Suppose the distribution of voters registered in one of the states in the U.S. is the following: 40% Democrats, 36% Republicans, 10% other parties, and the rest are independent. Suppose you select 3 people at random from this list of voters. Show steps and reasons, for finding the following probabilities [answers alone, whether correct or not, will receive no credit]

(a) No one is Democrat.
(b) The 3rd person is the first Republican you selected.
(c) At least one person is independent.

Solution

If we assume party affiliations are disjoint, we can deduce the individual probabilities:

\[ P(D) = 0.40, \quad P(R) = 0.36, \quad P(O) = 0.10, \quad P(I) = 1 - (0.40 + 0.36 + 0.10) = 0.14 \]

\[ D = \text{Democrat}, \quad R = \text{Republican}, \quad O = \text{Other parties}, \quad I = \text{independent} \]

For all questions, since the 3 individuals are randomly selected, their party affiliations should be independent. So we can use the simple multiplication rule for “AND” type probabilities.

(a) No one is D: This requires 3 back-to-back not D’s.

\[ P(\text{none of 3 is } D) = P(\sim D \text{ and } \sim D \text{ and } \sim D) = P(\sim D) \times P(\sim D) \times P(\sim D) \]

\[ P(\sim D) = 1 - 0.4 = 0.6, \quad \text{so we get:} \]

\[ P(\text{none of 3 is } D) = 0.6 \times 0.6 \times 0.6 \quad = 0.216 \]

(b) The 3rd person is the first Republican selected.

This can only happen in one way: The first 2 are not R, and the 3rd is R.

\[ P(\text{3rd person is first } R) = (1 - 0.36) \times (1 - 0.36) \times 0.36 \quad = 0.1475 \]

(c) At least one independent: There are many different ways in which this can occur. So, it is best to work with the complement of what the question asks for. Complement of “At least one independent” is “No one is independent,” which is very much like question (a) above.

\[ P(\text{none of 3 is } I) = P(\sim I) \times P(\sim I) \times P(\sim I) = 0.86 \times 0.86 \times 0.86 \]

\[ \text{[the last step comes from using } P(\sim I) = 1 - P(I) = 0.86]\]

\[ \text{Finally, } P(\text{at least one } I) = 1 - P(\text{none of 3 is } I) = 1 - 0.6361 \quad = 0.3639 \]

Grading: Total points possible = 10.

1 pt - Any reasonable attempt.
3 pt each for (a) (b) and (c): 1pt = correct steps
1pt = correct reasons
1pt = computations