

Student name:

MATH 120: Elementary Statistics
Spring 2022

Test 2
April 13, 2022

Instructions:

- This is a regular “closed-book” test, and is to be taken without the use of notes, books, or other reference materials. Collaboration or group work is not permitted.
 - Cell-phone usage of any kind is prohibited for the entire duration of the test. This also applies to any restroom breaks taken during the test.
 - The time limit for taking this test is 50 minutes from the scheduled start time. Please turn in your test promptly when time is called to avoid late penalties.
 - This test adds up to 50 points. It contains questions numbered 1 through 7.
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1. [4 points] Given the probabilities $P(U)=0.3$, $P(V)=0.6$, find the probability $P(U \text{ and } V)$ for each of the following situations:

U and V are disjoint

U and V are independent

2. [6 points] Given independent random variables X , Y , with means and standard deviations as shown, find the mean and SD of:

(a) $-2Y$

	Mean	SD
X	40	8
Y	100	12

(b) $X + Y$

3. [7 points] Public health officials in a state in the U.S. claim that 18% of adults currently smoke cigarettes. Assume that figure is true, and suppose we pick a random sample of 100 adults from that state and count how many of them smoke.
- (a) Under what assumptions can we treat this like a binomial experiment?

 - (b) Assume those conditions are met, define an appropriate random variable, and indicate which specific binomial probability distribution it follows.

 - (c) Suppose we want to compute the probability that exactly 25 adults in our sample of 100 smoke. Write an expression for computing that probability.
Hint: It should contain an appropriate form of ${}_nC_r$.

 - (d) Assume your binomial model can be approximated by an appropriate normal model. What is the correct normal model (e.g., its mean, SD)?

4. [7 points] An insurance policy costs \$150 annually, and will pay policyholders \$20,000 if they suffer major injury, or \$5000 if they suffer a minor injury. The company estimates that each year 1 in every 1000 policyholders will have a major injury and 1 in 500 will have a minor injury.
- (a) Create a probability model for the company's profit on a policy.
 - (b) Find the expected profit.
 - (c) Find the standard deviation of the profit.

5. [7 points] The distribution of voters registered in a state is the following: 40% Democrat, 36% Republican, 10% other parties, and the rest are independent. Suppose 3 people are selected randomly from this list, find the probability that
- (a) None of the 3 is a Democrat.
 - (b) The group includes at least one independent.
 - (c) Exactly one of the 3 is Republican.

Be sure to show calculation steps and state any assumptions you make.

6. [7 points] The career services office at a small liberal arts college has compiled data on the career path of their recent graduating class. The following 2-way table shows these data, organized by type of career path chosen by graduates in different fields of study.

	Major area of study				Total
	Fine arts	Humanities	Natural sci.	Social sci.	
Employed	17	41	43	62	163
Grad school	9	19	23	25	76
Total	26	60	66	87	239

- (a) Find the probability that a randomly selected graduate is employed or majored in the natural sciences.
- (b) What is the probability that an individual who went to graduate school majored in the fine arts?
- (c) Are majoring in the humanities and going to graduate school disjoint, independent, neither, or both?

[Solution must show key steps and correct interpretation of each question - e.g., probability of employed and social science, or $P(\text{employed and social science})$, etc.]

7. [4 points \times 3] Give brief solutions to the following (unrelated) questions as instructed. It is always helpful to include a relevant sketch, and to show key calculation steps.

(a) Every normal distribution is defined by its parameters, the mean μ , and the standard deviation σ . Suppose for a particular normal distribution we know $\mu = 7$, and 5% of the distribution lies below the value of 2. Find σ .

(b) Let X be a normally distributed random variable with $\mu = 30$ and $\sigma = 10$.

(i) Find the probability that $X \geq 54$.

(ii) Find the *IQR* for this distribution.

(c) Suppose scores on the math section of the SAT follow the distribution $N(525, 120)$.

(i) Find the z -score for an SAT score of 300.

(ii) Scores on the ACT math test, which is an alternative to the SAT, follow the normal model $N(21, 5.5)$. Suppose person A scored 600 on the math SAT, and person B scored 25 on the ACT math test. Whose performance was better? Why?