MATH 120: Quiz 8 - 4/29/2022

A consumer advocacy group is interested in estimating the mean credit card debt per household in Indiana. They select a random sample of 64 households in Indiana and find the mean and standard deviation of credit card debt is \$15,355 and \$10,000, respectively.

Compute and interpret a 90% confidence interval to estimate the mean credit card debt per household in Indiana. (Make sure your solution includes all needed steps: check conditions, sampling distribution model, computations, and conclusion.)

Solution

In this problem, the sample statistics are: $\bar{x} = \$15,355$ and $s_x = \$10,000$.
The sample size is: $n = 64$.

Checking the conditions for applying the Central Limit Theorem:

(i) Is the sample independent: For this, must check randomness, and whether $n < 10\%$ of the population.
Randomness: The question states that the sample of 64 households is random.
Is $n < 10\%$: Yes, 64 should be less than 10% of all households in Indiana.
(ii) Is the population approximately normally distributed?
No relevant information is provided. But, unless the population is severely skewed, a sample size of 64 is large enough to safely proceed.

All the conditions for applying inference procedures appear to be satisfied.

The sampling distribution follows the student-t model $t_{63}(\mu, \frac{10000}{\sqrt{64}})$ dollars.

$$\text{Margin of Error} = t^*_{63}\frac{s_x}{\sqrt{n}} = (1.671)\frac{10000}{\sqrt{64}} = \$2088.75$$

(The value of t^*_{63} is taken from the t-table for 90% confidence with $df = 60$.)

Therefore, the confidence interval is: $\bar{x} \pm ME = 15,355 \pm 2088.75$.

Answer: The 90% confidence interval $= [13266.25, 17443.75]$.
Conclusion: With 90% confidence, the true mean credit card debt per household in Indiana is between \$13,266 and \$17,444.

Grading: Total points possible = 6.

- 1+0.5pt = know + check correct 2 conditions.
- 1.5pt = correct model details: t, df, SE.
- 0.5pt each for correct t^* and ME.
- 1pt = compute correct confidence interval.
- 1pt = state correct conclusion.