

MATH 120: Quiz 7 - 4/08/2022

A card game: You draw a card at random from a deck. If you get a red card, you win nothing. If you get a spade, you win \$7. For any club, you win \$15, plus an extra \$25 for the ace of clubs.

- Create a probability model for the amount you win.
- Find the expected amount you'll win.
- Find the standard deviation of the amount.

(Be sure to show steps/reasons.)

Solution

- Let X = random variable that represents the amount I win.

The possible values of X are \$40, \$15, \$7, 0.

The probability model is shown in the table below:

X (in \$)	40	15	7	0
$P(X)$	1/52	12/52	13/52	26/52

Reason: $P(40)=1/52$ because there is only one ace of clubs in 52 cards, and the prize for it is \$40. $P(15)=12/52$ because there are 12 remaining clubs, for which the prize is \$15. $P(7)=13/52$ because the prize for spades is \$7 and there are 13 spades out of 52. $P(0)=26/52$ because there is no prize for red cards and there are 26 red cards.

$$(b) E(X) = \sum x \cdot P(x) = 40 \cdot (1/52) + 15 \cdot (12/52) + 7 \cdot (13/52) + 0 \cdot (26/52)$$

$$\boxed{E(X) = \$5.98} = \bar{x}$$

$$(c) SD(X) = \sqrt{\sum (x - \bar{x})^2 P(x)}$$

$$= \sqrt{\frac{(40 - 5.98)^2}{52} + \frac{(15 - 5.98)^2 \cdot 12}{52} + \frac{(7 - 5.98)^2 \cdot 13}{52} + \frac{(0 - 5.98)^2 \cdot 26}{52}}$$
$$= \sqrt{59.1727} = \$7.6924$$

Answers:

(b) The expected amount of winnings = \$5.98
(c) The standard deviation of winnings = \$7.6924

Grading: Total points possible = 6.

2.5 points each for (a) and (b); 1 point for (c).

For (a): 1 pt = correct values of X ; 1.5 pt = correct values of $P(X)$.

For (b): 1 pt = show correct plug into formula; 1.5 pt = compute answer (with units).

For (c): 0.5+0.5 pt = plug into correct formula + compute correct answer.