MATH 120: Quiz 6 - 4/01/2022

Suppose you are playing a game that involves rolling a fair 6-sided die, with the following rules:

- You must pay \$5 to play.
- If you roll a 6, you win \$20.
- If you roll a 4 or 5, you win \$5.
- If you roll any other number, you win nothing.

Assume we are interested in modeling your net gain (or loss), inclusive of the cost of playing. E.g., if you win nothing, your net gain is $-\$5$.

(a) In words, define the random variable.

(b) Create a probability model for this variable.

(c) Compute the expected value.

~~(d) Compute the standard deviation.~~

(Be sure to show steps/reasons.)

Solution

(a) Since we're interested in my net gain (or loss), the random variable is

X = the net amount (in \$) I gain or lose from playing the game.

(b) There are 3 possible values of X: \$15 (if I roll a 6)
\$0 (if I roll 4 or 5)
$-\$5$ (any other roll)

Thus, the probability model is

X (in \$)	15	0	-5
$P(x)$	1/6	2/6	3/6

Reason: probability of a 6 = 1/6, probability of 4 or 5 = 2/6, and probability of any other number = 3/6

(c) $E(X) = \sum x \cdot P(x) = 15 \cdot \frac{1}{6} + 0 \cdot \frac{2}{6} - 5 \cdot \frac{3}{6} = 0$

Thus, the expected value of my net gain is $\boxed{0 \text{ dollars}}$

(d) $SD(X) = \sqrt{\sum (x - \bar{x})^2 \cdot P(x)}$, where $\bar{x} = E(x)$.

Since $\bar{x} = 0$ in this problem, the calculations become easier, and we get

$$SD(X) = \sqrt{15^2 \cdot \frac{1}{6} + 0^2 \cdot \frac{2}{6} + (-5)^2 \cdot \frac{3}{6}} = \sqrt{\frac{225}{6} + \frac{75}{6}} = 7.071$$

The standard deviation of my net gain is $\boxed{7.071 \text{ dollars}}$

Grading: Total points possible = 6.

1 point for (a); 2.5 points each for (b) and (c).

For (a): 1pt for correct answer; -0.5 if sloppy words leave "net" gain unclear.

For (b): 1pt for correct values of X; 1.5pt for correct (matching) values of $P(x)$.
-0.5pt if X values represent "gross" gain, instead of "net" gain.

For (c): 0.5pt = attempt to plug into the right formula for $E(X)$
1.5pt = do the plugin correctly.
0.5pt = get answer.
NOTE: no further penalty for this part, even if X represents "gross" gain.

~~For (d): 0.5pt=correctly plug into formula for SD.~~
~~0.5pt=get correct answer.~~