Probability worksheet

1. Given the probabilities $P(B) = 0.5$, $P(C) = 0.6$, and $P(B|C) = 2/3$:
   (a) Find the probability $P(B \text{ and } C)$. Show steps.
   (b) Find the probability $P(B \text{ or } C)$.
   (c) In this problem, events B and C are not independent. Can you explain why?

2. Every normal distribution is defined by its parameters, the mean μ, and the standard deviation σ. Suppose for a particular normal distribution we know $\sigma = 120$, and 10% of the distribution lies below the value of 60. Find μ.

3. The Masterfoods company says the colors of plain M&M candies are made in the following proportions: yellow=20%, red=20%, orange, blue and green are each 10%, and the rest are brown. Suppose you pick 3 M&Ms at random. Show steps and reasons for finding the following probabilities [*answers alone, whether correct or not, will receive no credit*]
   (a) The first M&M is the only red that you get.
   (b) At least one M&M is yellow.
   (c) None of the M&Ms is brown.
   (d) The 3rd M&M is the first time you get a brown.
   (e) You get all 3 M&Ms of the same color.

4. In one of your recent homework problems you were given the following scenario: "A soccer team estimates that they will score on 8% of their corner kicks."

   Suppose we want to compute probabilities for the number of times they will score on their next 200 corner kicks.

   (a) Under what assumptions can we treat this like a binomial experiment?
   (b) Assume those conditions are met, define an appropriate random variable, and indicate which specific binomial probability distribution it follows.
   (c) Assume this binomial model can be approximated by an appropriate normal model. What is the appropriate normal model?
   (d) Use the normal model to estimate the probability that the team will score on at least 25 of those 200 corner kicks.

5. In one city in the Midwest it is estimated that 70% of teenagers who go to take the written drivers license test have studied for it. Of those who study for the test, 95% pass. Of those who don't study, 60% pass the test.
   (a) Find the probability that a randomly selected teenager taking the drivers license test in that city passes.
   (b) Find the probability that a teenager who passes the test did not study for it.
   (c) What is the probability that a teenager passes the test and did not study for it?
   (d) Are studying for the test and passing it disjoint, independent, neither, or both? [Show details of work and reasoning.]

6. A bag contains 6 pennies, 4 dimes and 2 quarters, randomly mixed together. Suppose you draw 2 coins from this bag, what is the probability that both are the same type of coin (penny, dime, quarter)? Show calculation steps.

7. A 2014 Pew Research Center survey on re-marriage trends in the U.S. asked a group of divorced adults to answer the question: "In general, what's your preference? Do you want to get married again, don't you want to get married again, or are you not sure?" The 2-way table shows the distribution of responses by sex:

	Response to remarriage question			
	Yes	Not sure	No	**Total**
Female	450	902	1768	3120
Male	306	382	324	1012
Total	756	1284	2092	4132

(a) Find the probability that a random person from this sample responded "No."
(b) Find the probability that someone who responded "No" was female.
(c) What is the probability that a female did not respond "No?"
(d) What is the probability that 3 randomly selected persons all responded "Yes?"
[*Be sure to restate each question in correct probability notation, and show calculation steps for full credit.*]

Just FYI, a more advanced version of question (d) might be: What is the probability that 3 randomly selected persons gave the same response?

Or: What is the probability that at least one of 3 randomly selected persons responded "Yes?"

8. A normally distributed random variable has mean= -47 and standard deviation= 10.
(a) Find the probability that its value is larger than -25.
(b) Find Q_1, Q_3, and the IQR for this distribution.

Show all calculations and relevant sketches.

9. An auto rental company's fleet consists of 70% U.S. brands, and the rest are foreign brands. The company notes that manufacturers' recalls seem to affect about 1% of the U.S. brands, and about 0.5% of the foreign brands.

(a) Find the probability that a randomly selected vehicle in their fleet is recalled.

(b) Find the probability that a recalled vehicle is a foreign brand.

(c) What is the probability that a vehicle is recalled and it is a foreign brand.

[Show details of work & reasoning.]

10. A survey of real estate ads shows that 65% of homes for sale have garages, 20% have swimming pools, and 13% have both. Show steps and sketch or table for finding the following probabilities [*answers alone, whether correct or not, will receive no credit*]:

(a) A randomly selected home for sale has a garage or a simming pool.

(b) A home for sale that has a garage, also has a swimming pool.
   Hint: The answer is not 0.13.

(c) Of two randomly selected homes, at least one has a garage.

(d) Are having a garage and a pool statistically independent?
   Use the probability-based definition of independence to check.

11. Suppose the distribution of voters registered in one of the states in the U.S. is the following: 48% Democrats, 42% Republicans, and the rest are from other parties or independent. Suppose you select 3 people at random from this list of voters. Find the probability of each of the following events:
   (a) The 3rd person is the first Republican you selected.
   (b) At least one person is Democrat.
   (c) None of the 3 is from "the other" parties.
   (d) All 3 are from the same party, either Democratic or Republican (ignore others).

12. A service company bids on two contracts and knows it can only get one of them. It anticipates a profit of $200,000 if it gets the larger contract, and a profit of $40,000 on the smaller contract. The company estimates it has a 30% chance of getting the larger contract, and a 65% chance of getting the smaller contract.

   (a) Create a probability model for the company's anticipated profit.
   (b) Find the expected profit.
   (c) Find the standard deviation of the profit.

Selected answers

(With 90% probability that they are correct!)

1. (a) $P(B \text{ and } C) = 0.4$.
   (b) $P(B \text{ or } C) = 0.7$.
   (c) Because $P(B) \neq P(B|C)$.

2. $\mu = 213.6$.

3. (a) $P(\text{only first one is R}) = (0.2)(0.8)^2$
   (b) $P(\text{at least one Y}) = 1 - (0.8)^3$
   (c) $P(\text{none brown}) = (0.7)^3$
   (d) $P(\text{third M\&M is first brown}) = (0.7)^2(0.3)$
   (e) $P(\text{all same color}) = 2(0.2)^3 + 3(0.1)^3 + (0.3)^3$

4. (a) Assume each kick is independent and has the same probability of success.
   (b) Let $X = $ the number of times they score on their next 200 corner kicks.
       Then $X \sim B(200, 0.08)$.
   (c) $N(16, 3.8367)$
   (d) $P(X \geq 25) = 0.0095$

5. (a) $P(\text{Pass}) = 0.845$
   (b) $P(\text{Not studied} \mid \text{Pass}) = 0.213$
   (c) $P(\text{Not studied and Pass}) = 0.18$
   (d) Neither disjoint, nor independent.

6. $P(\text{Both coins of same kind}) = 0.333$

8. (a) 0.0139.
   (b) $Q_1 = -53.8, \ Q_3 = -40.2, \ IQR = 13.6$

9. (a) $P(\text{Recalled}) = 0.0085$
   (b) $P(\text{Foreign} \mid \text{Recalled}) = 0.1765$
   (c) $P(\text{Foreign and Recalled}) = 0.0015$

10. (a) $P(\text{Garage or Pool}) = 0.72$
    (b) $P(\text{Pool} \mid \text{Garage}) = 0.2$
    (c) $P(\text{at least one of two random homes has Garage}) = 0.8775$
    (d) Yes, the events are independent.

11. (a) $P(\text{the 3rd person is the first Republican}) = 0.1413$
    (b) $P(\text{at least one is Dem}) = 1 - (0.52)^3 = 0.8594$
    (c) $P(\text{none from other parties}) = 0.729$
    (d) $P(\text{all 3 from same party, D or R}) = 0.1847$

12. (b) Expected profit $= \$86,000$
    (c) SD of profit $= \$75,126.56$