

Probability Assignment Rule	The probability of the entire sample space must be 1. $P(S) = 1$ (p. 327).
Complement Rule	The probability of an event not occurring is 1 minus the probability that it does occur (p. 327). $P(A^c) = 1 - P(A)$
Addition Rule	If A and B are disjoint events, then the probability of A or B is (p. 327) $P(A \text{ or } B) = P(A) + P(B).$
Disjoint (mutually exclusive)	Two events are disjoint if they share no outcomes in common. If A and B are disjoint, then knowing that A occurs tells us that B cannot occur. Disjoint events are also called “mutually exclusive” (p. 327).
Legitimate assignment of probabilities	An assignment of probabilities to outcomes is legitimate if (p. 328) <ul style="list-style-type: none"> ■ each probability is between 0 and 1 (inclusive). ■ the sum of the probabilities is 1.
Multiplication Rule	If A and B are independent events, then the probability of A and B is (p. 328) $P(A \text{ and } B) = P(A) \times P(B).$

Exercises

Section 12.1

- Flipping a coin** Flipping a fair coin is said to randomly generate heads and tails with equal probability. Explain what random means in this context.
- Dice** Rolling a fair six-sided die is supposed to randomly generate the numbers 1 through 6. Explain what random means in this context.
- Flipping a coin II** Your friend says: “I flipped five heads in a row! The next one has to be tails!” Explain why this thinking is incorrect.
- Dice II** After rolling doubles on a pair of dice three times in a row, your friend exclaims, “I can’t get doubles four times in a row!” Explain why this thinking is incorrect.

Section 12.2

- Wardrobe** In your dresser are five blue shirts, three red shirts, and two black shirts.
 - What is the probability of randomly selecting a red shirt?
 - What is the probability that a randomly selected shirt is not black?
- Playlists** Your list of favorite songs contains 10 rock songs, 7 rap songs, and 3 country songs.
 - What is the probability that a randomly played song is a rap song?
 - What is the probability that a randomly played song is not country?

Section 12.3

- Cell phones and surveys** A 2010 study conducted by the National Center for Health Statistics found that 25% of U.S. households had no landline service. This raises concerns about the accuracy of certain surveys, as they depend on random-digit dialing to households via landlines. We are going to pick five U.S. households at random:
 - What is the probability that all five of them have a landline?
 - What is the probability that at least one of them does not have a landline?
 - What is the probability that at least one of them does have a landline?
- Cell phones and surveys II** The survey by the National Center for Health Statistics further found that 49% of adults ages 25–29 had only a cell phone and no landline. We randomly select four 25–29-year-olds:
 - What is the probability that all of these adults have a only a cell phone and no landline?
 - What is the probability that none of these adults have only a cell phone and no landline?
 - What is the probability that at least one of these adults has only a cell phone and no landline?

Chapter Exercises

- Sample spaces** For each of the following, list the sample space and tell whether you think the events are equally likely:
 - Toss 2 coins; record the order of heads and tails.
 - A family has 3 children; record the number of boys.
 - Flip a coin until you get a head or 3 consecutive tails; record each flip.
 - Roll two dice; record the larger number (or simply the number in case of a tie).

10. **Sample spaces** For each of the following, list the sample space and tell whether you think the events are equally likely:

- a) Roll two dice; record the sum of the numbers.
- b) A family has 3 children; record each child's sex in order of birth.
- c) Toss four coins; record the number of tails.
- d) Toss a coin 10 times; record the length of the longest run of heads.

11. **Roulette** A casino claims that its roulette wheel is truly random. What should that claim mean?

12. **Rain** The weather reporter on TV makes predictions such as a 25% chance of rain. What do you think is the meaning of such a phrase?

13. **Winter** Comment on the following quotation:

"What I think is our best determination is it will be a colder than normal winter," said Pamela Naber Knox, a Wisconsin state climatologist. "I'm basing that on a couple of different things. First, in looking at the past few winters, there has been a lack of really cold weather. Even though we are not supposed to use the law of averages, we are due." (Source: Associated Press, fall 1992, quoted by Schaeffer et al.)

14. **Snow** After an unusually dry autumn, a radio announcer is heard to say, "Watch out! We'll pay for these sunny days later on this winter." Explain what he's trying to say, and comment on the validity of his reasoning.

15. **Cold streak** A basketball player missed 8 of 11 shots in the final portion of a game. When talking to reporters afterward, he says that he's not worried about his next game because he's due to make a lot of shots. Comment on his reasoning.

16. **Crash** Commercial airplanes have an excellent safety record. Nevertheless, there are crashes occasionally, with the loss of many lives. In the weeks following a crash, airlines often report a drop in the number of passengers, probably because people are afraid to risk flying.

- a) A travel agent suggests that since the law of averages makes it highly unlikely to have two plane crashes within a few weeks of each other, flying soon after a crash is the safest time. What do you think?
- b) If the airline industry proudly announces that it has set a new record for the longest period of safe flights, would you be reluctant to fly? Are the airlines due to have a crash?

17. **Auto insurance** Insurance companies collect annual payments from drivers in exchange for paying for the cost of accidents.

- a) Why should you be reluctant to accept a \$1500 payment from your neighbor to cover his automobile accidents in the next year?
- b) Why can the insurance company make that offer?

18. **Jackpot** On February 11, 2009, the AP news wire released the following story:

(LAS VEGAS, Nev.)—A man in town to watch the NCAA basketball tournament hit a \$38.7 million jackpot on Friday, the biggest slot machine payout ever. The 25-year-old software engineer from Los Angeles, whose name was not released at his request, won after putting three \$1 coins in a machine at the Excalibur hotel-casino, said Rick Sorensen, a spokesman for slot machine maker International Game Technology.

- a) How can the Excalibur afford to give away millions of dollars on a \$3 bet?
- b) Why was the maker willing to make a statement? Wouldn't most businesses want to keep such a huge loss quiet?

19. **Spinner** The plastic arrow on a spinner for a child's game stops rotating to point at a color that will determine what happens next. Which of the following probability assignments are possible?

Probabilities of . . .				
	Red	Yellow	Green	Blue
a)	0.25	0.25	0.25	0.25
b)	0.10	0.20	0.30	0.40
c)	0.20	0.30	0.40	0.50
d)	0	0	1.00	0
e)	0.10	0.20	1.20	-1.50

20. **Scratch off** Many stores run "secret sales": Shoppers receive cards that determine how large a discount they get, but the percentage is revealed by scratching off that black stuff (what is that?) only after the purchase has been totaled at the cash register. The store is required to reveal (in the fine print) the distribution of discounts available. Which of these probability assignments are legitimate?

Probabilities of . . .				
	10% Off	20% Off	30% Off	50% Off
a)	0.20	0.20	0.20	0.20
b)	0.50	0.30	0.20	0.10
c)	0.80	0.10	0.05	0.05
d)	0.75	0.25	0.25	-0.25
e)	1.00	0	0	0

21. **Electronics** Suppose that 46% of families living in a certain county own a computer and 18% own an HDTV. The Addition Rule might suggest, then, that 64% of families own either a computer or an HDTV. What's wrong with that reasoning?

22. **Homes Funding** For many schools comes from taxes based on assessed values of local properties. People's homes are assessed higher if they have extra features such as garages and swimming pools. Assessment records in a certain school district indicate that 37% of the homes have garages and 3% have swimming pools. The Addition Rule might suggest, then, that 40% of residences have a garage or a pool. What's wrong with that reasoning?

- 23. Speeders** Traffic checks on a certain section of highway suggest that 60% of drivers are speeding there. Since $0.6 \times 0.6 = 0.36$, the Multiplication Rule might suggest that there's a 36% chance that two vehicles in a row are both speeding. What's wrong with that reasoning?
- 24. Lefties** Although it's hard to be definitive in classifying people as right- or left-handed, some studies suggest that about 14% of people are left-handed. Since $0.14 \times 0.14 = 0.0196$, the Multiplication Rule might suggest that there's about a 2% chance that a brother and a sister are both lefties. What's wrong with that reasoning?
- 25. College admissions** For high school students graduating in 2007, college admissions to the nation's most selective schools were the most competitive in memory (Source: *The New York Times*, "A Great Year for Ivy League Schools, but Not So Good for Applicants to Them," April 4, 2007). Harvard accepted about 7% of its applicants, MIT 10%, and Penn 14%. Jorge has applied to all three. Assuming that he's a typical applicant, he figures that his chances of getting into both Harvard and MIT must be about 0.7%.
- How has he arrived at this conclusion?
 - What additional assumption is he making?
 - Do you agree with his conclusion?
- 26. College admissions II** In Exercise 25, we saw that in 2007 Harvard accepted about 7% of its applicants, MIT 10%, and Penn 14%. Jorge has applied to all three. He figures that his chances of getting into at least one of the three must be about 31%.
- How has he arrived at this conclusion?
 - What assumption is he making?
 - Do you agree with his conclusion?
- 27. Car repairs** A consumer organization estimates that over a 1-year period 17% of cars will need to be repaired only once, 7% will need repairs exactly twice, and 4% will require three or more repairs. What is the probability that a car chosen at random will need
- no repairs?
 - no more than one repair?
 - some repairs?
- 28. Stats projects** In a large Introductory Statistics lecture hall, the professor reports that 55% of the students enrolled have never taken a Calculus course, 32% have taken only one semester of Calculus, and the rest have taken two or more semesters of Calculus. The professor randomly assigns students to groups of three to work on a project for the course. What is the probability that the first groupmate you meet has studied
- two or more semesters of Calculus?
 - some Calculus?
 - no more than one semester of Calculus?
- 29. More repairs** Consider again the auto repair rates described in Exercise 27. If you own two cars, what is the probability that
- neither will need repair?
 - both will need repair?
 - at least one car will need repair?
- 30. Another project** You are assigned to be part of a group of three students from the Intro Stats class described in Exercise 28. What is the probability that of your other two groupmates,
- neither has studied Calculus?
 - both have studied at least one semester of Calculus?
 - at least one has had more than one semester of Calculus?
- 31. Repairs again** You used the Multiplication Rule to calculate repair probabilities for your cars in Exercise 29.
- What must be true about your cars in order to make that approach valid?
 - Do you think this assumption is reasonable? Explain.
- 32. Final project** You used the Multiplication Rule to calculate probabilities about the Calculus background of your Statistics groupmates in Exercise 30.
- What must be true about the groups in order to make that approach valid?
 - Do you think this assumption is reasonable? Explain.
- 33. Energy 2011** A Gallup Poll in March 2011 asked 1012 U.S. adults whether increasing domestic energy production or protecting the environment should be given a higher priority. Here are the results:

Response	Number
Increase Production	511
Protect Environment	419
Equally Important	41
No Opinion	41
Total	1012

If we select a person at random from this sample of 1012 adults,

- what is the probability that the person responded "Increase production"?
- what is the probability that the person responded "Equally important" or had no opinion?

- 34. Failing fathers?** A Pew Research poll in 2011 asked 2005 U.S. adults whether being a father today is harder than it was a generation ago. Here's how they responded:

Response	Number
Easier	501
Same	802
Harder	682
No Opinion	20
Total	2005

- If we select a respondent at random from this sample of 2005 adults,
- what is the probability that the selected person responded "Harder"?
 - what is the probability that the person responded the "Same" or "Easier"?
35. **More energy** Exercise 33 shows the results of a Gallup Poll about energy. Suppose we select three people at random from this sample.
- What is the probability that all three responded "Protect the environment"?
 - What is the probability that none responded "Equally important"?
 - What assumption did you make in computing these probabilities?
 - Explain why you think that assumption is reasonable.
36. **Fathers, revisited** Consider again the results of the poll about fathering discussed in Exercise 34. If we select two people at random from this sample,
- what is the probability that both think that being a father is easier today?
 - what is the probability that neither thinks being a father is easier today?
 - what is the probability that the first person thinks being a father is easier today and the second one doesn't?
 - what assumption did you make in computing these probabilities?
 - explain why you think that assumption is reasonable.
37. **Polling** As mentioned in the chapter, opinion-polling organizations contact their respondents by sampling random telephone numbers. Although interviewers can reach about 76% of U.S. households, the percentage of those contacted who agree to cooperate with the survey fell from 58% in 1997 to only 38% in 2003 (Source: Pew Research Center for the People and the Press). Each household, of course, is independent of the others. Using the cooperation rate from 2003,
- what is the probability that the next household on the list will be contacted but will refuse to cooperate?
 - what is the probability of failing to contact a household or of contacting the household but not getting them to agree to the interview?
 - show another way to calculate the probability in part b.
38. **Polling, part II** According to Pew Research, the contact rate (probability of contacting a selected household) was 69% in 1997 and 76% in 2003. However, the cooperation rate (probability of someone at the contacted household agreeing to be interviewed) was 58% in 1997 and dropped to 38% in 2003.
- What is the probability (in 2003) of obtaining an interview with the next household on the sample list? (To obtain an interview, an interviewer must both contact the household and then get agreement for the interview.)
 - Was it more likely to obtain an interview from a randomly selected household in 1997 or in 2003?
39. **M&M's** The Mars company says that before the introduction of purple, yellow candies made up 20% of their plain M&M's, red another 20%, and orange, blue, and green each made up 10%. The rest were brown.
- If you pick an M&M at random, what is the probability that
 - it is brown?
 - it is yellow or orange?
 - it is not green?
 - it is striped?
 - If you pick three M&M's in a row, what is the probability that
 - they are all brown?
 - the third one is the first one that's red?
 - none are yellow?
 - at least one is green?
40. **Blood** The American Red Cross says that about 45% of the U.S. population has Type O blood, 40% Type A, 11% Type B, and the rest Type AB.
- Someone volunteers to give blood. What is the probability that this donor
 - has Type AB blood?
 - has Type A or Type B?
 - is not Type O?
 - Among four potential donors, what is the probability that
 - all are Type O?
 - no one is Type AB?
 - they are not all Type A?
 - at least one person is Type B?
41. **Disjoint or independent?** In Exercise 39, you calculated probabilities of getting various M&M's. Some of your answers depended on the assumption that the outcomes described were *disjoint*; that is, they could not both happen at the same time. Other answers depended on the assumption that the events were *independent*; that is, the occurrence of one of them doesn't affect the probability of the other. Do you understand the difference between disjoint and independent?
- If you draw one M&M, are the events of getting a red one and getting an orange one disjoint, independent, or neither?
 - If you draw two M&M's one after the other, are the events of getting a red on the first and a red on the second disjoint, independent, or neither?
 - Can disjoint events ever be independent? Explain.
42. **Disjoint or independent?** In Exercise 40, you calculated probabilities involving various blood types. Some of your answers depended on the assumption that the outcomes described were *disjoint*; that is, they could not both happen at the same time. Other answers depended on the assumption that the events were *independent*; that is, the occurrence of one of them doesn't affect the probability

of the other. Do you understand the difference between disjoint and independent?

- a) If you examine one person, are the events that the person is Type A and that the same person is Type B disjoint, independent, or neither?
- b) If you examine two people, are the events that the first is Type A and the second Type B disjoint, independent, or neither?
- c) Can disjoint events ever be independent? Explain.
- 43. Dice** You roll a fair die three times. What is the probability that
- a) you roll all 6's?
- b) you roll all odd numbers?
- c) none of your rolls gets a number divisible by 3?
- d) you roll at least one 5?
- e) the numbers you roll are not all 5's?
- 44. Slot machine** A slot machine has three wheels that spin independently. Each has 10 equally likely symbols: 4 bars, 3 lemons, 2 cherries, and a bell. If you play, what is the probability that
- a) you get 3 lemons?
- b) you get no fruit symbols?
- c) you get 3 bells (the jackpot)?
- d) you get no bells?
- e) you get at least one bar (an automatic loser)?
- 45. Champion bowler** A certain bowler can bowl a strike 70% of the time. If the bowls are independent, what's the probability that she
- a) goes three consecutive frames without a strike?
- b) makes her first strike in the third frame?
- c) has at least one strike in the first three frames?
- d) bowls a perfect game (12 consecutive strikes)?
- 46. The train** To get to work, a commuter must cross train tracks. The time the train arrives varies slightly from day to day, but the commuter estimates he'll get stopped on about 15% of work days. During a certain 5-day work week, what is the probability that he
- a) gets stopped on Monday and again on Tuesday?
- b) gets stopped for the first time on Thursday?
- c) gets stopped every day?
- d) gets stopped at least once during the week?
- 47. Voters** Suppose that in your city 37% of the voters are registered as Democrats, 29% as Republicans, and 11% as members of other parties (Liberal, Right to Life, Green, etc.). Voters not aligned with any official party are termed "Independent." You are conducting a poll by calling registered voters at random. In your first three calls, what is the probability you talk to
- a) all Republicans?
- b) no Democrats?
- c) at least one Independent?
- 48. Religion** Census reports for a city indicate that 62% of residents classify themselves as Christian, 12% as Jewish, and 16% as members of other religions (Muslims, Buddhists, etc.). The remaining residents classify themselves as nonreligious. A polling organization seeking information about public opinions wants to be sure to talk with people holding a variety of religious views, and makes random phone calls. Among the first four people they call, what is the probability they reach
- a) all Christians?
- b) no Jews?
- c) at least one person who is nonreligious?
- 49. Lights** You purchased a five-pack of new light bulbs that were recalled because 6% of the lights did not work. What is the probability that at least one of your lights is defective?
- 50. Pepsi** For a sales promotion, the manufacturer places winning symbols under the caps of 10% of all Pepsi bottles. You buy a six-pack. What is the probability that you win something?
- 51. 9/11?** On September 11, 2002, the first anniversary of the terrorist attack on the World Trade Center, the New York State Lottery's daily number came up 9-1-1. An interesting coincidence or a cosmic sign?
- a) What is the probability that the winning three numbers match the date on any given day?
- b) What is the probability that a whole year passes without this happening?
- c) What is the probability that the date and winning lottery number match at least once during any year?
- d) If every one of the 50 states has a three-digit lottery, what is the probability that at least one of them will come up 9-1-1 on September 11?
- 52. Red cards** You shuffle a deck of cards and then start turning them over one at a time. The first one is red. So is the second. And the third. In fact, you are surprised to get 10 red cards in a row. You start thinking, "The next one is due to be black!"
- a) Are you correct in thinking that there's a higher probability that the next card will be black than red? Explain.
- b) Is this an example of the Law of Large Numbers? Explain.



Just Checking ANSWERS

- The LLN works only in the long run, not in the short run. The random methods for selecting lottery numbers have no memory of previous picks, so there is no change in the probability that a certain number will come up.
- 0.76
 - $(0.76)(0.76) = 0.5776$
 - $(1 - 0.76)^2(0.76) = 0.043776$
 - $1 - (1 - 0.76)^5 = 0.9992$