Homework due April 18

Assigned exercises:
From Ch.7, OpenStax book, ex. 62, 63, 66, 68, 75(e,f,g), 83, 84, 91, 96.
From linked supplement: 39a, 40, 50. (total=12 numbered exercises)

Graded exercises:
From Ch.7, OpenStax book: 62, 75, 91, 96.
From linked supplement: 40.

Total (maximum) possible points = 20.
3 pt for each of 5 graded problem sets, plus 5 for completion of the rest.
−0.5 pt for each (ungraded) missing problem; if a graded problem is missing, student loses the points allotted to it.

Exercises from Ch.7, OpenStax

(62) (a) $\bar{x} \sim N(250, \frac{50}{\sqrt{49}})$ feet.

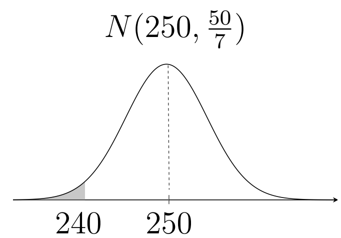
This assumes the conditions for the CLT are met, which is reasonable here since: (1) the sample likely satisfies independence, since it is random, and (2) a sample size of 49 is large enough, since the population is normal.

(b) To find the probability that $\bar{x} < 240$ feet:

$$z = \frac{240 - 250}{50/7} = -1.4.$$

From z-table, $P(z < -1.4) = 0.0808$

Answer: The probability that the mean distance traveled is less than 240' = 0.0808



(c) From z-table, 80th percentile occurs at $z = 0.84$.

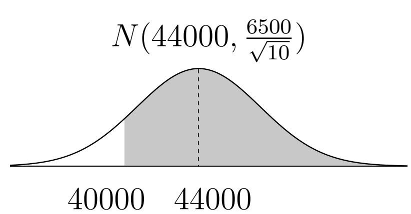
$\therefore\ \bar{x} = (0.84)(50/7) + 250 \ \boxed{= 256 \text{ feet}}$

Grade: 1.5 pt. each for (a) and (b). (c) is not graded.
For (a): 0.5pt each for 3 items: (i) N, (ii) mean=250, (iii) sd=$50/\sqrt{49}$.
For (b): 0.5 pt each for 3 items: (i) find z-score, (ii) find probability, (iii) sketch.
Notes: Sketch doesn't have to be perfect, but an attempt is required. There is no penalty, but a reprimand for not checking the CLT conditions in part (a).

(75) (e) The probability that $\sum X > \$400,000$ is the same as the probability the mean $> \frac{400,000}{10} = \$40000$.

Since the sample is random and drawn from a normal population, we can apply the CLT and get $\bar{X} \sim N(44000, 6500/\sqrt{10})$

$$z = \frac{40000 - 44000}{(6500/\sqrt{10})} = -1.95.$$



From z-table, $P(z > -1.95) = 1 - 0.0256 = 0.9744$

Answer: The probability that the teachers earn a total of more than \$40,000 = 0.9744.

(f) The relevant model for an individual teacher's salary is $N(44000, 6500)$.

From z-table, 90th percentile occurs at $z = 1.28$.

$\therefore$ 90th percentile salary = (1.28)(6500) + 44000 $\boxed{= \$52320}$

(g) The model for mean salary of 10 teachers is $N(44000, \frac{6500}{\sqrt{10}})$.

$\therefore$ 90th percentile of mean salary $= (1.28)(\frac{6500}{\sqrt{10}}) + 44000 = 46631.015$

Thus, the 90th percentile of the sum of 10 teachers salaries $\boxed{= \$466,310.15}$

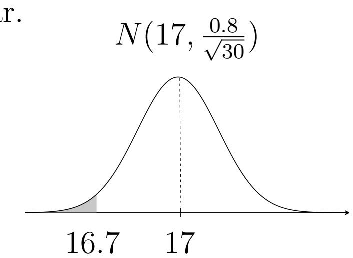
Grade: 1.5 pt. each for (e) and (f). (g) is not graded.
For (e): 0.5pt each for 3 items: (i) correct mean and sd of normal model, (ii) correct z-score, (iii) correct probability.
For (f): 0.5pt each for: (i) mean/sd of model, (ii) z score, (iii) salary computation.

(91) To use sampling distribution model based on the CLT, let's first check the conditions:

(1) Is the sample independent? Yes, this would be reasonable to assume, since it was randomly selected and 30 batteries is likely less than 10% of all batteries produced by the company.

(2) Is the sample large enough? $n = 30$ would be large enough if the population distribution (lifespan of all batteries) is not severely skewed. But that information is not known.

Summary: Conditions may be met, but it is not 100% clear.



The relevant model for mean lifespan (say, $\bar{x}$) of samples of 30 batteries is: $N(17, \frac{0.8}{\sqrt{30}})$ hours.
To find the probability that $\bar{x} < 16.7$ hours:

$$z = \frac{16.7 - 17}{(0.8/\sqrt{30})} = -2.054.$$

From z-table, $P(z < -2.054) = 0.0202$

The probability that the mean lifespan of a sample of 30 batteries is below 16.7 hours is 0.0202. Since this is a very low probability, the company's claim that the population mean is 17 hours is questionable.

Grade: 0.5 + 0.5pt = check each CLT condition (even sloppy/poorly done is ok)
1 pt = compute correct z score.
0.5 pt = look up correct probability.
0.5 pt = state a reasonable conclusion – either company's claim is dubious because ..., or their claim is okay because ....

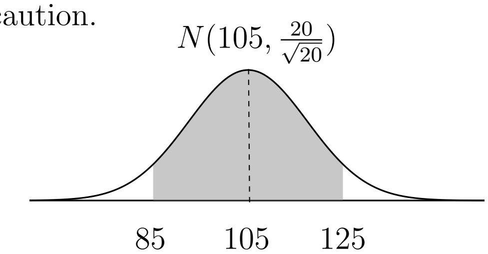
(96) Check the conditions for CLT:

(1) Is the sample independent? Yes, this is reasonable to assume, since it was randomly selected and 20 adults is less than 10% of all adults.

(2) Is the sample large enough? $n = 20$ may not be large enough, unless the population distribution (IQ scores of all adults) is close to normal. Since that information is not given, this condition may not be met. Proceed with caution.

The relevant model for the mean IQ score (say, $\bar{x}$) of samples of 20 adults is: $N(105, \frac{20}{\sqrt{20}})$.
To find the probability that $85 < \bar{x} < 125$:

$$z_1 = \frac{85 - 105}{(20/\sqrt{20})} = -4.47, \quad z_2 = \frac{125 - 105}{(20/\sqrt{20})} = 4.47.$$



Since these z scores are over 4σ on either side of the mean, the probability is essentially 1. In other words, the mean IQ score of a random sample of 20 adults is guaranteed to be between 85 and 125.

Grade: 0.5pt = check CLT conditions (even sloppy/poorly done is ok)
1 pt = state correct model (esp. mean and sd)
1 pt = compute two correct z scores.
0.5 pt = state a reasonable conclusion – e.g., mean IQ scores has high probability of being between 85 and 125.

Exercises from linked supplement

(40) (a) If a small sample is drawn from a skewed population, the sampling distribution of mean values is likely to have a similar skew. So, if the population is strongly left-skewed, the sampling distribution will also be left-skewed.

(b-c) As the sample size gets larger, the sampling distribution of mean values will approach a normal shape, with mean approaching the population's mean, and SD approaching $\sigma/\sqrt{n}$. This is guaranteed by the Central Limit Theorem.

Grade: (a)=1 point, (b) and (c) together = 2 points.
For (a): any answer that recognizes the sampling distribution will not have a normal shape is good.
For (b) and (c) together:
1pt=any answer that recognizes the shape will approach a normal curve.
1pt = say something reasonable about the mean and sd – e.g., mean will be close to the population's mean, and the sd will get smaller.