Homework due April 22

Assigned exercises:

From linked supplement: 27, 30, 32, 33, 35, 36. (total=6 numbered exercises)

Graded exercises:

From linked supplement: 27, 32, 36.

Total (maximum) possible points = 20.

5 pt for each of 3 graded problems, plus 5 for completion of the rest.

−1.5 pt for each (ungraded) missing problem; if a graded problem is missing, student loses the points allotted to it.

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(27) (a) Assuming the Central Limit Theorem (CLT) is applicable here, the sampling distribution of the proportion of customers who might not make timely payments will have: mean=0.07, standard deviation= $\sqrt{(0.07 \times 0.93)/200} = 0.018$

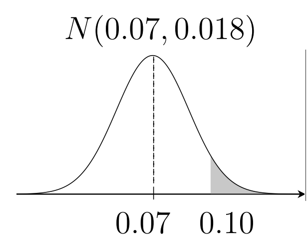
(b) The assumptions are: (1) the sample is made up of independent observations; and (2) the sample size is large enough.

It is not clear whether (1) is satisfied, since we are only told that the sample consists of 200 "recently approved" loans. Unless they were randomly selected, or are at least representative, the sample may not be independent.

The sample does satisfy (2), since: $n \cdot p = (200)(0.07) = 14$ and $n \cdot (1 - p) = 186$, both larger than the minimum requirement of 10.

(c) Here we will need to use the CLT, in spite of our reservations about the conditions. The model is

$$\hat{p} \sim N(0.07, 0.018) \text{ feet.}$$



To find the probability that $\hat{p} > 0.1$:

$$z = \frac{0.10 - 0.07}{0.018} = 1.663.$$

From z-table, $P(z > 1.663) = 0.0485$

Answer: The probability that more than 10% will not make timely payments = 0.0485. We note, however, that this conclusion may not be reliable, as it is not clear whether the sample satisfies the independence requirement.

Grade: (a) = 1 point, (b) and (c) =2 points each.
For (a): 0.5+0.5 pt for correct mean + correct sd.
For (b): 1+1 pt for checking independence + sample size condition. Doesn't have to be perfect, or even 100% correct, but an attempt is required.
For (c): 1 pt = compute correct z-score;
1 pt = correctly lookup probability and state the answer.

(32) Since the conditions for applying the CLT have already been checked in exercise (30), we will proceed with the assumption that they are satisfied.

In this problem we have: $p = 0.44$ (the national rate of binge drinking).
$n = 244$ (the size of the sample).
$\hat{p} = 96/244 = 0.3934$ (rate of binge drinking in sample).

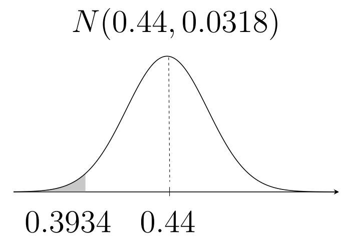
The sampling distribution model is:

$$N(0.44, \sqrt{\tfrac{(0.44)(0.56)}{244}})$$

To find the probability that $\hat{p} \leq 0.3934$:

$$z = \frac{0.3934 - 0.44}{0.0318} = -1.465.$$

From z-table, $P(z \leq -1.465) = 0.071$



Answer: The professor should not be too surprised about the lower binge drinking rate in his sample, because there is a 7% probability it is simply due to sampling variability. In other words, the true binge drinking rate on his campus may well be the same as the national rate of 44%.

> **Grade**:
> 0.5+0.5 pt = know correct shape (i.e., normal) + mean of sampling dist model.
> 1pt = compute correct SD of sampling dist model.
> 0.5 pt = know / compute correct value of $\hat{p}$
> 1+1 pt = compute z-score + get correct probability (via table lookup or software).
> 0.5 pt = write a reasonable conclusion.
> (Note: Student's conclusion need not agree with mine. It is okay to say that "7% is a low probability and so, yes, the professor should be surprised.")

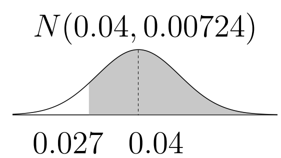
(36) To use sampling distribution model based on the CLT, let's first check the conditions:

(1) Is the sample independent? Nothing in the problem indicates whether it is random, or at least representative of the population of interest. It is not clear whether the independence condition is met.

(2) Is the sample large enough? We have $n = 732$ and $p = 0.04$. Thus, $n \cdot p = 29.3$ and $n \cdot (1 - p) = 702.7$, both larger than 10. Thus, the sample is large enough.

The relevant model is: $N(0.04, \sqrt{\tfrac{(0.04)(0.96)}{732}})$

The researchers want to find at least 20 cases in the sample. Thus, $\hat{p} = 20/732 = 0.027$



To find the probability that $\hat{p} \geq 0.027$: $z = \dfrac{0.027 - 0.04}{0.00724} = -1.75$

From z-table, $P(z \geq -1.75) = 0.96$

The probability that the researchers will find enough subjects for their study is about 96%. We note, however, that this conclusion may not be reliable, as it is not clear whether the sample satisfies the independence requirement.

Grade:
1pt = reasonable check of conditions for CLT.
0.5+0.5 pt = find correct mean + SD of sampling dist model.
0.5 pt = know / compute correct value of $\hat{p}$
1+1 pt = compute z-score + get correct probability (via table lookup or software).
0.5 pt = write a reasonable conclusion.
(Note: Student's conclusion need not agree with mine. It is okay to say that "7% is a low probability and so the professor should surprised.")