Theorem 3.5: Let A be a set, and let $\mathcal{S}$ be a partition of A. There is an equivalence relation $\sim$ on A such that $\mathcal{S} = A/\sim$.

Solution: I will use a direct proof.
{* *State my hypotheses*}
(1) Let A be a set, and let $\mathcal{S}$ be a partition of A.
{* *Outline my strategy*}
(2) I will define a relation $\sim$ on A, and then show
(i) $\sim$ is an equivalence relation on A, and
(ii) $\mathcal{S} = A/\sim$
{* *Define* $\sim$}
(3) Let $\sim$ be a relation on A such that for all $x, y \in A$:
$x \sim y$ iff there exists $P \in \mathcal{S}$ such that $x, y \in P$.
{* *Show it is an equivalence relation*}
(4) I'll now show that $\sim$ is an equivalence relation on A.
(4.1) Consider any $x \in A$.
Since $\mathcal{S}$ is a partition of A, $x \in P$ for some $P \in \mathcal{S}$ [by definition of partition]
This means $x \sim x$, since $x, x \in P$ [by definition of $\sim$ in line (3)]
It follows that $\sim$ is reflexive.
(4.2)

(Student proves it is symmetric and transitive).

(5) Lines (4.1)-(4.3) show that $\sim$ is an equivalence relation on A.

{* *Next, show* $\mathcal{S} = A/\sim$}
(6) Instead of proving $\mathcal{S} = A/\sim$, I'll show that each equivalence class of A under $\sim$ coincides with some set in the partition $\mathcal{S}$.
[My instructor agreed to accept that as a substitute for proving $\mathcal{S} = A/\sim$]

(7) Accordingly, let $x \in A$, and let E_x denote the equivalence class of x under $\sim$.
(8) Since $x \in A$, there is some $T \in \mathcal{S}$ such that $x \in T$, because _________________________.
(9) I will prove that $E_x = T$ by establishing a subset relation in both directions.
(9.1) Proof of $E_x \subseteq T$.
(9.1.1) Let $m \in E_x$. I must show that $m \in T$.

(Student completes the argument).

(9.2) Proof of $T \subseteq E_x$.

(6.2.1) Let $n \in T$. I must show that $n \in E_x$.

(Student completes the argument).

(9.3) The results from (9.1) and (9.2) together prove that $E_x = T$.

(9.4) Therefore, we have shown there exists an equivalence relation $\sim$ on A induced by the partition $\mathcal{S}$. Furthermore, every equivalence class of A coincides with some piece of the partition. This completes the proof.