Homework due date Aug. 23

Assigned exercises: 1.7: 6, 8, 12, 13, 14, 18, 21, 24, 34.
2.3: 2, 4. 2.4: 1, 3. (13 problems)
Graded exercises: 1.7: 12, 18, 21. 2.3: 4. 2.4: 3.
Total (maximum) possible points = 20.
3 pt for each of 5 graded problems, plus 5 for completion of the rest.

Exercise 1.7

(12) Given: There exists a natural number x such that for every integer y, $x - y$ is a natural number.
This is false, because for any $x \in \mathbb{N}$ there exists $-(x+1) = y \in \mathbb{Z}$, which means $x + y = -1$ and this is not in $\mathbb{N}$.
Negation: For every natural number x there exists an integer y such that $x - y$ is not a natural number.
In symbols: $(\forall x \in \mathbb{N})\ (\exists y \in \mathbb{Z})\ ((x - y) \notin \mathbb{N})$

(18) Given: For all real numbers x and y, if $x \leq y$ and $x \geq y$ then $x = y$.
True. For any $x, y \in \mathbb{R}$ there are only 3 possibilities: $x < y$ or $x = y$ or $x > y$. If $x \leq y$ and $x \geq y$ is true, then $x = y$ is the only possibility.
Negation: There exist real numbers x and y for which $x \leq y$ and $x \geq y$, but $x \neq y$.
In symbols: $(\exists\, x, y \in \mathbb{R})((x \leq y) \wedge (x \geq y) \wedge (x \neq y))$

(21) Given: There exists $x \in \mathbb{N}$ such that for all $y \in \mathbb{N}$, $x \neq 2y$ and $x \neq 2y - 1$.
False. Any $x \in \mathbb{N}$ is either even or odd. If it is even, then let $y = x/2$. If it is odd, then let $y = (x+1)/2$. In both cases y exists, and it is in $\mathbb{N}$.
Negation: For every $x \in \mathbb{N}$ there exists $y \in \mathbb{N}$ such that $x = 2y$ or $x = 2y - 1$.
In symbols: $(\forall x \in \mathbb{N})\ (\exists y \in \mathbb{N})\ ((x = 2y) \vee (x = 2y - 1))$.

Exercise 2.3

(4) Prove: If $A \cap C \subseteq B \cap C$ and $A \cup C \subseteq B \cup C$ then $A \subseteq B$.
Solution:
(a) Let A, B, C be sets such that $A \cap C \subseteq B \cap C$ and $A \cup C \subseteq B \cup C$.
(b) I want to prove $A \subseteq B$. By way of contradiction, suppose $A \nsubseteq B$.
(c) Then, $\exists x \in A$ such that $x \notin B$. [by negating defn of subset]
(d) Since $x \in A$, we know $x \in A \cup C$. [by defn of union]
(e) Then $x \in B \cup C$. [from hypothesis $A \cup C \subseteq B \cup C$]
(f) This implies $x \in B$ or $x \in C$. [by defn of union]
(g) Consider each possibility: If $x \in B$, then we have a contradiction with line (c), which says $x \notin B$.

(h) On the other hand, if $x \in C$, then $x \in A \cap C$. [since $x \in A$ from line (c)]

(i) This implies $x \in B \cap C$. [since $A \cap C \subseteq B \cap C$ by hypothesis]

(j) Then $x \in B$. [by defn of intersection]

(k) Again, we have a contradiction with line (c), which says $x \notin B$.

(l) Since every case leads to a contradiction, our supposition that $A \nsubseteq B$ is impossible.

(m) It follows that if $A \cap C \subseteq B \cap C$ and $A \cup C \subseteq B \cup C$, then $A \subseteq B$.

Remark: This solution is somewhat simpler if proved directly. But a contradiction based proof has been presented here following the authors' instructions.

Exercise 2.4

(3) Prove: If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

Solution:

(a) Let A, B, C be sets such that $A \subseteq B$ and $B \subseteq C$.

(b) To prove $A \subseteq C$, suppose $m \in A$.

(c) Then $m \in B$. [since $A \subseteq B$]

(d) This implies $m \in C$. [since $B \subseteq C$]

(e) From line (b), (d), we have: $m \in A \Rightarrow m \in C$.

(f) It follows that $A \subseteq C$.