Homework due date Aug. 18

Assigned exercises: 1.5: 5-8. 1.6: 6, 7, 8. 1.7: 1-4. (11 problems)
Graded exercises: 1.5: 6, 7. 1.6: 7. 1.7: 3, 4.
Total (maximum) possible points = 20.
3 pt for each of 5 graded problems, plus 5 for completion of the rest.

Exercise 1.5

(6) Given: $(\exists y \in \mathbb{N})\ (\forall x \in \mathbb{N})\ (x = y - 7)$

In Words: There exists a natural number y such that for all natural numbers x, $x = y - 7$.
This is false, because we want a single, fixed y to satisfy $y = x + 7$ for all natural numbers x. Unless y is allowed to change with x, the equation cannot possibly hold for all x.

(7) Given: $(\forall y \in \mathbb{N})\ (\exists x \in \mathbb{N})\ (x = y - 7)$

In Words: For every natural number y there exists a natural number x such that $x = y - 7$.
This is false. Although the statement is true for *most* natural numbers y, it doesn't hold when $y \leq 7$.

Exercise 1.6

(7) Rewrite in symbols: There is no largest integer.

Answer: $(\forall x \in \mathbb{Z})\ (\exists y \in \mathbb{Z})\ (y > x)$

Exercise 1.7

(3) Given: $(\forall x \in \mathbb{Z})\ ((x > 0) \Rightarrow (x \in \mathbb{N}))$

This statement is true, because every integer larger than 0 is a natural number.
Negation: There exists a positive integer that is not a a natural number.
In symbols: $(\exists x \in \mathbb{Z})\ ((x > 0) \wedge (x \notin \mathbb{N}))$

(4) Given: $(\forall x \in \mathbb{N})\ ((x > 2) \Rightarrow (x > 3))$

This statement is false, because it doesn't hold when $x = 3$. For that case, the implication has true hypothesis and false conclusion.
Negation: There exists a natural number that is greater than 2, but not greater than 3.
In symbols: $(\exists x \in \mathbb{N})\ ((x > 2) \wedge (x \leq 3))$