A warmup

1. Let A be the set $\{1, 2, 3\}$. Consider the sets $S = \{1\}, T = \{2, 3\}$.

   (a) What is $S \cup T$ and $S \cap T$?

   (b) Show that $\{S, T\}$ is a partition of A.

   (c) This partition corresponds to a (unique) equivalence relation on set A. An easy way to find this relation is to assume S, T are its equivalence classes.

2. Let $A = \{p, q, r, s, t, u, v, w\}$, and $\mathcal{S} = \{\{p, q, r, s\}, \{t, u\}, \{v, w\}\}$.

   (a) What is $\bigcup \mathcal{S}$ and $\bigcap \mathcal{S}$?

   (b) Check whether $\mathcal{S}$ satisfies all the conditions for being a partition of A.

   (c) Find the equivalence relation that corresponds to $\mathcal{S}$.

Moral of the story:
Every partition of a set induces an equivalence relation on it.

Key Theorems

(NOTE: These are informal, non-technical statements of the theorems.)

(1) Any pair of equivalence classes (of an equivalence relation on a set) is either identical, or disjoint.

(2a) Every equivalence relation on a set induces a partition on it.

AND

(2b) Every partition of a set induces an equivalence relation on it.

Proof strategy for (2a):

1. Consider the set $\mathcal{S}$ of equivalence classes of $\boldsymbol{r}$.
2. Show that $\mathcal{S}$ satisfies all the definitions of a partition.

   (a) Is every $X \in \mathcal{S}$ non-empty?

   (b) Is $\bigcup_{X \in \mathcal{S}} X = A$?

   (c) Is $X_1 \cap X_2 = \emptyset$ whenever $X_1 \neq X_2$?

Proof strategy for (2b):

1. Let set $\mathcal{S}$ denote some partition of A.
2. Define a relation $\sim$ on A such that $x \sim y$ iff x and y belong to the same "piece" of the partition $\mathcal{S}$.
3. Show that $\sim$ is reflexive, symmetric, and transitive.