

A warmup

1. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$.

Define the functions $f : A \rightarrow B$ and $g : B \rightarrow A$ as follows

$$f = \{(1, a), (2, b), (3, a)\} \text{ and } g = \{(a, 1), (b, 3), (c, 2)\}$$

Describe each of the following functions by listing its ordered pairs:

$$(a) g^{-1} \quad (b) f \circ g \quad (c) g \circ f \quad (d) f \circ g \circ f$$

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Find $f^{-1}(A)$ for each of the following:

(a) $A = \{9\}$

(b) $A = [4, 9)$

(c) $A = [-4, 9)$

(d) $A = \{x \in \mathbb{R} \mid -9 < x \leq 0\}$

3. For each of the following, find a function $f : \mathbb{N} \rightarrow \mathbb{N}$ with the indicated properties:

(a) f is bijective

(b) f is injective, but not surjective

(c) f is surjective, but not injective