Brief glimpse into math profession & culture - 2

There are some famous open (unsolved) problems in math. It is useful to know a little bit about them because they have significantly impacted various aspects of math culture – e.g., areas of research, how we teach certain topics, folklore and stories in math, etc.

Hilbert Problems

These consist of 23 major math problems proposed by Hilbert in the year 1900 at the International Congress of Mathematicians in Paris. These problems span a broad range of areas within mathematics (and beyond!), and have profoundly shaped mathematical study and research to this day. More than 100 years later, some of these problems remain unsolved. Here are some websites with more information:

- `https://mathcs.clarku.edu/~djoyce/hilbert/`
- `https://www.cmi.ac.in/~smahanta/hilbert.html`
- `https://mathworld.wolfram.com/HilbertsProblems.html`
- `https://kids.kiddle.co/Hilbert's_problems`

The Millennium Prize Problems

These consist of 7 "Prize Problems" proposed by the The Clay Mathematics Institute (CMI) of Cambridge, Massachusetts, in the year 2000. Each problem carries a prize of $1 million for its solution. According to the CMI, "The prizes were conceived to record some of the most difficult problems with which mathematicians were grappling at the turn of the second millenium." One of them was solved in 2006, and six remain unsolved. For more information:

- `https://www.claymath.org/millennium-problems/millennium-prize-problems`
- `https://brilliant.org/wiki/millennium-prize-problems/`

A warmup

Let $S = \{4, 5, 6\}$ and suppose $\boldsymbol{r}$ denotes an equivalence relation on S.

1. Find the $\boldsymbol{r}$ that has the fewest members. How many distinct equivalence classes exist for this $\boldsymbol{r}$? Describe the corresponding partition of S.

2. Find the $\boldsymbol{r}$ with the fewest members such that $(4, 5)$ is in $\boldsymbol{r}$. How many distinct equivalence classes exist for this $\boldsymbol{r}$? Describe the corresponding partition of S.

3. Find the $\boldsymbol{r}$ with the fewest members such that $(4, 5)$ and $(5, 6)$ are in $\boldsymbol{r}$. How many distinct equivalence classes exist for this $\boldsymbol{r}$? Describe the corresponding partition of S.