Test practice worksheet

1. Write negations for each of the following:

   (a) If x is an integer, then there is no rational number y such that $y^2 = x$.

   (b) If A, B, C, D are sets such that $A \subseteq B$ and $C \subseteq D$, then $A \cup C \subseteq B \cup D$.

   (c) If $x + y$ is even and $y + z$ is even, then $x + z$ is even.

   (d) If f is a continuous function, then it is both differentiable and integrable.

   (e) If f is a continuous function, then it is differentiable, integrable, squarable and filterable.

   (f) If f is a continuous and increasing function on $\mathbb{R}$, then $f(x) > f(y)$ whenever $x > y$.

   (g) If f is a differentiable and increasing function on $\mathbb{R}$, then $f(x) > f(y)$ and $f'(x) > f'(y)$ whenever $x > y$.

   (h) If a person eats an apple a day, it will keep the doctor away.

   (i) If n is a positive integer it can be written as the sum of distinct powers of 2, or as the sum of distinct powers of 3.

   (j) The roots of this polynomial are either all real or all complex.

2. Write the contrapositive of each statement above.

3. Rephrase each statement in Q1 above in a form that does not involve "If ... then." For example, "If x is a natural number then it is positive" can be written as, "All natural numbers are positive."

4. Consider the statements:

$P :=$ Dogs eat meat
$Q :=$ Rome is in Italy
$R :=$ Chocolate prevents cavities
$S :=$ The moon is made of green cheese

Based on the known, real-world truth value of these statements, determine whether each of the following is true or false:

(a) If P, then Q (b) If P, then R (c) If R, then S

(d) If S, then Q (e) If Q, then S