Some logic warmups

1. Consider the theorem: "If m^2 is odd, then m is odd." What, if anything, is wrong with each of the following "proofs."

   (a) Suppose m is odd. Then by definition $m = 2k + 1$ for some integer k. Thus $m^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$, which is odd. Thus, we've shown if m^2 is odd, then m is odd.

   (b) Suppose m is not odd. Then m is even, and by definition $m = 2k$ for some integer k. Thus $m^2 = (2k)^2 = 4k^2 = 2(2k^2)$, which is even. This implies, if m is not odd then m^2 is not odd. It follows that if m^2 is odd, then m is odd.

2. Consider the theorem: "If x is rational and y is irrational, then xy is irrational." Consider the following proof:

   Suppose x is rational and y is irrational.
   By way of contradiction, suppose xy is rational.
   Then, by definition, $x = p/q$ and $xy = m/n$ for some integers p, q, m, n, with $q \neq 0$ and $n \neq 0$. This implies

   $$y = \frac{xy}{x} = \frac{m/n}{p/q} = \frac{mq}{np}$$

   Since mq and np are integers, this implies y is rational, which is a contradiction. Thus we've proved if x is rational and y is irrational, then xy is irrational.

   (a) Find a counterexample that proves this theorem is false.

   (b) What is/are the errors in the above proof?

   (c) We can make the theorem true by adding some condition to the hypothesis. What is that condition?

Q: Consider the sets:

$$A = \{3, 0, 6, 3, 5, 3, 0, 6\}$$
$$B = \{0, 3, 5, 6\}$$
$$C = \{6, 5, 3, 0\}$$
$$D = \{3, 6, 0, 5\}$$
$$E = \{0, 0, 0, 3, 3, 3, 3, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6\}$$

Something seems "fishy" about these sets. What? Is there some relationship between them? Discuss some thoughts and ideas.

Ans:

They are all the same set!

Can you prove $A = B = C = D = E$?

Moral of the story:

Neither order, nor duplication of elements makes any difference to a set.