Negation: Introductory exercises

Write a negation of each of the following in words and in symbols (where possible).

1. Words: Every natural number is rational.
   Symbols: $(\forall m \in \mathbb{N})\ (m \in \mathbb{Q})$ OR $(m \in \mathbb{N}) \Rightarrow (m \in \mathbb{Q})$

2. Words: There exists a natural number that is rational.
   Symbols: $(\exists m \in \mathbb{N})\ (m \in \mathbb{Q})$

3. Words: Every real number is either rational or irrational.
   Symbols: $(\forall x \in \mathbb{R})\ ((x \in \mathbb{Q}) \vee (x \in (\mathbb{R} - \mathbb{Q})))$
   OR $(x \in \mathbb{R}) \Rightarrow ((x \in \mathbb{Q}) \vee (x \in (\mathbb{R} - \mathbb{Q})))$

4. Words: Every interval of real numbers contains rational numbers and irrational numbers.
   Symbols: <leave for later>

5. Words: Every interval of real numbers contains rational numbers or irrational numbers.
   Symbols: <leave for later>

6. Words: There exists an interval of real numbers that contains rational numbers and irrational numbers.
   Symbols: <leave for later>

7. Words: For any positive real number x, there exists a positive real number y such that $y^2 = x$.
   Symbols: $(\forall x \in \mathbb{R}^+)\ \ ((\exists y \in \mathbb{R}^+)\ \ni (\ y^2 = x))$

8. Words: There exists a positive real number y such that for all positive real numbers x, $y^2 = x$.
   Symbols: $(\exists y \in \mathbb{R}^+)\ \ni\ (\forall x \in \mathbb{R}^+,\ y^2 = x)$