A warmup

Anand is having this conversation with a student.

Anand: Hey, guess what - I've just discovered a cool new theorem! It says: "Every integer that is a perfect square is even."

Student: Wow, that's cool! Can you prove it?

Anand: Sure! Here's a proof:

> Consider the number 4, which is a perfect square, since $4 = 2^2$. Clearly, 4 is even by definition, since $4 = 2 \times 2$.
>
> Next consider the number 16, which is a perfect square, because $16 = 4^2$. By definition, it is even, since $16 = 2(8)$.
>
> By similar argument I can show that the numbers 36, 64, 100, 900, 2500, and infinitely many others are perfect squares, and they are all even.
>
> It follows that every perfect square is even.

What is the moral of the story?!!!

Class exercises

Rewrite each of the following in two different forms:

(i) Using only/primarily mathematical symbols.

(ii) In the form "If p, then q."

1. The sum of any two rational numbers is rational.
2. For any positive real number x, there exists a positive real number y such that $y^2 = x$.
3. The product of any two odd integers is odd.
4. The sum of a rational number and an irrational number, is irrational.
5. For every integer x, either x is a natural number or $-x$ is a natural number.
6. There exist integers l, m, n, such that $l^2 + m^2 = n^2$.

LaTeX exercises

Rewrite each of the following in two different forms in LaTeX:

(i) Using only/primarily mathematical symbols.

(ii) In the form "If p, then q."

1. The sum of any two rational numbers is rational.
2. For any positive real number x, there exists a positive real number y such that $y^2 = x$.
3. The product of any two odd integers is odd.
4. The sum of a rational number and an irrational number, is irrational.