Polynomial Functions: Key Aspects

General form

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \bullet\bullet\bullet + a_n x^n$$

(a_0, a_1, a_2, etc., are all numbers)

Role model polynomials

Quadratic: $f(x) = x^2$; Cubic: $f(x) = x^3$

The highest-power term of a polynomial

The term $a_n x^n$ is key that unlocks many "secrets" of the polynomial:

* It tells us "the order" of the polynomial --> order=n
* How many zeros (i.e., x-intercepts) the polynomial can have
* How many U-turns the polynomial can have

Why the order is helpful to know

Think of the highest power term by itself: $a_n x^n$
If n is even, function is even & graph goes upward for large $\pm x$ (quadratic role model). If n is odd, the function is odd & graph $\uparrow$ for large $+x$, $\downarrow$ for $-x$.

Coefficient of the highest power (a_n)

If a_n is positive and n is even, graph goes $\uparrow$ for large $\pm x$.

If a_n is positive and n is odd, graph goes $\uparrow$ for $+x$, and $\downarrow$ for $-x$.

Q: What happens if a_n is negative in the above cases?

Zeros of a polynomial

Can find them by factoring, or by locating the x-intercepts.

How many zeros

A polynomial can have at most n zeros.

Can have fewer than n if there are "repeated" zeros, or complex zeros.

Repeated zeros, if they exist, show up when factoring. They lead to repeated factors (i.e., factors raised to an integer power).

Steps in making a rough graph of a polynomial

(0) Turn off your graphing calculator!

(1) Determine left-end & right-end behavior
[for $x \to \pm\infty$, is $f(x)$ going up or down?]

(2) Find the zeros.

(3) Make a sign-chart of behavior between zeros.

(4) Plug in a few select x-values & evaluate $f(x)$.

(5) Make rough graph.

Key features of graphs of polynomials

* Continuous curve, with no breaks or pieces.

* No vertical or horizontal asymptotes.

* On both sides (positive & negative x), graph eventually goes off to $\pm\infty$ in the y-direction.

Asymptotes of rationals: Recap

Vertical

Graph goes off to ∞ in the vertical direction -- either up or down.
Follows a vertical line (e.g., x=c) in doing so.
Often "returns" from ∞ on other side of this line.
Occurs at x-values where the denominator = 0.

A function can have more than 1 vertical asymptote.

Horizontal

Graph goes off to ∞ in the horizontal direction -- either left or right.
Follows a horizontal line (e.g., y=c) in doing so.
Rational functions can have at most 1 horizontal asymptote.

To find the y-value where it occurs, for $f(x) = \dfrac{N(x)}{D(x)}$:

Locate the term that has highest power in x:

(1) If this term is in $N(x)$, there is no horizontal asymptote.

(2) If this term is in $D(x)$, the horizontal asymptote is the x-axis.

(3) If the highest power is the same in $N(x)$ and $D(x)$ then find the ratio of coefficients for this term. This gives the y-value of the horizontal asymptote.