Algebraic methods: Strategy summary

To find $\lim\limits_{x \to a} f(x)$ [we assume $f(x)$ is NOT a piecewise function]:

- Try to plug $x = a$ into $f(x)$. If it gives a finite value, that is the limit.

- If plugin fails, do the following
  [essentially, we want to try to cancel out $x - a$, and then plugin again]

  ➢ Factor out $x - a$ from numerator and denominator. Cancel & plugin $x = a$.

  ➢ Do some algebra, e.g., common denominator, simplify, etc. Look for an $x - a$ cancellation, then try plugin again.

  ➢ If $f(x)$ has square root terms, try to rationalize – multiply numerator and denominator by "conjugate" of the square root expression. Then simplify and look for an $x - a$ cancellation. Then plugin $x = a$.
  E.g., the conjugate of $(\sqrt{x} + 1)$ is $(\sqrt{x} - 1)$; of $(2x - \sqrt{x - 3})$ is $(2x + \sqrt{x - 3})$; of $(\sqrt{1 - 2x} + \sqrt{x + 3})$ is $(\sqrt{1 - 2x} - \sqrt{x + 3})$.

Exercises (from old tests):

- $\lim\limits_{x \to -1} \dfrac{x + 1}{1 + \dfrac{1}{x}}$

- $\lim\limits_{x \to -1} \dfrac{x + 1}{\sqrt{x + 5} - 2}$

- $\lim\limits_{x \to -1} \dfrac{x^3 + x^2 - x - 1}{x + 1}$

- $\lim\limits_{x \to 1} \left(\dfrac{1}{x - 1} + \dfrac{2}{1 - x^2} \right)$