(I) Negate the following:

1. For all real numbers r, $r^2 \geq 0$.
2. For all real numbers $p \in S$, if p is prime then $p \in U$.
3. There exists a rational number q whose square is 2.
4. There exists an integer n which, when added to 3, yields -1.
5. For each real number r there is a real number s such that $rs = 1$.
6. If n is an even integer, then n^2 is an even integer.
7. If $a < b$ and c is positive, then $ac < bc$.
8. If $x > 0$ or $x < 0$, then $x^2 > 0$.
9. If $a < b$, then there exists a c such that $a < c < b$.
10. If m and n are positive integers, then so are $m + n$ and mn.
11. Given positive rational numbers y and z, there is a positive integer n such that $ny > z$.
12. Given cuts α and β, the equation $\alpha = \beta x$ has a unique solution for x.

(II) Write the contrapositive of each of the following statements:

1. If n is an even integer, then n^2 is an even integer.
2. If $a < b$ and c is positive, then $ac < bc$.
3. If $x > 0$ or $x < 0$, then $x^2 > 0$.
4. If $a < b$, then there exists a c such that $a < c < b$.
5. If m and n are positive integers, then so are $m + n$ and mn.

(III) Rewrite each of the following as an implication, and then write its contrapositive:

1. For all real numbers r, $r^2 \geq 0$.
2. For all real numbers $p \in S$, if p is prime then $p \in U$.
3. For each real number r there is a real number s such that $rs = 1$.
4. Given positive rational numbers y and z, there is a positive integer n such that $ny > z$.
5. Given cuts α and β, the equation $\alpha = \beta x$ has a unique solution for x.

Selected Solutions

(I) Negate the following:

2. For all real numbers $p \in S$, if p is prime then $p \in U$.
   Solution: There exists a prime number $p \in S$ such that $p \notin U$.

4. There exists an integer n which, when added to 3, yields -1.
   Solution: For every integer n, $n + 3$ does not equal -1.
   OR $n + 3$ does not equal -1 for any integer n.

6. If n is an even integer, then n^2 is an even integer.
   Solution: There exists an even integer n such that n^2 is an odd integer.

8. If $x > 0$ or $x < 0$, then $x^2 > 0$.
   Solution: There exists $x \neq 0$ such that $x^2 \leq 0$.
   OR There exists x such that $x > 0$ or $x < 0$, and $x^2 \leq 0$.

10. If m and n are positive integers, then so are $m + n$ and mn.
    Solution: There exist positive integers m and n such that either $m + n$ or mn is not a positive integer.

12. Given cuts α and β, the equation $\alpha = \beta x$ has a unique solution for x.
    Solution: There exist cuts α and β such that the equation $\alpha = \beta x$ does not have a unique solution for x.

(II) Write the contrapositive of each of the following statements:

2. If $a < b$ and c is positive, then $ac < bc$.
   Solution: If $ac \geq bc$, then either $a \geq b$ or c is non-positive.

4. If $a < b$, then there exists a c such that $a < c < b$.
   Solution: If for all c either $c \leq a$ or $c \geq b$, then $a \geq b$.
   OR If there is no c such that $a < c < b$, then $a \geq b$.

(III) Rewrite each of the following as an implication, and then write its contrapositive:

2. For all real numbers $p \in S$, if p is prime, then $p \in U$.
   Solution: If the real number $p \in S$ is prime, then $p \in U$.
   Contrapositive: If the real number p is not in U, then p is not in S or it is not a prime number.

4. Given positive rational numbers y and z, there is a positive integer n such that $ny > z$.
   Solution: If y and z are positive rational numbers, then there is a positive integer n such that $ny > z$.
   Contrapositive: If for all positive integers n, $ny \leq z$, then either y or z is not a positive rational number.