

## Big Picture View of Fundamental Theorem's Role in Calculus

### The tangent problem:.

Find the slope of the line tangent to the graph of  $f(x)$  at  $x = a$ .

### Approx. solution to tangent problem:

Find the slope of a secant line that is close to the tangent line of interest.

### Exact solution to tangent problem:

Take limit of the slope of secant lines, as the distance bet. its 2 points goes to 0.

⇒ Definition of derivative  $f'(x)$  at  $x = a$ .

### The area problem:

Find the area under the graph of  $f(x)$  between  $x = a$  and  $x = b$ .

### Approx. solution to area problem:

Tile the area of interest with rectangles whose heights follow the graph of  $f(x)$ .

### Exact solution to area problem:

Take limit of the area of rectangles, as the number of rectangles goes to infinity.

⇒ Definition of definite integral  $\int_a^b f(x) dx$

⇒ General area function  $\int_a^x f(t) dt$

### Fundamental Theorem of Calculus:.

(I) Derivative of area function =  $f(x)$  OR  $\frac{d}{dx} \int_a^x f(t) dt = f(x)$

(II) Evaluation of a definite integral:  $\int_a^b f(x) dx = F(b) - F(a)$ ,

where  $F(x)$  is any antiderivative of  $f(x)$ .