Worksheet

(I) Differentiate and simplify:

1. $y = e^{-2t}\cos(4t)$
2. $f(x) = e^{x\sin^2 x}$
3. $f(t) = \sqrt{\dfrac{t}{t^2+4}}$
4. $g(x) = \sqrt{\sin(\cos(x))}$
5. $f(t) = e^{\sin^2(4t+1)}$

(II) Find dy/dx for each of the following:

1. $x = \cos(t), \quad y = t\sin(t)$
2. $x = \theta - \sin^2(\theta), \quad y = \tan(2\theta)$
3. $x = \sqrt[3]{t^3-1}, \quad y = t/(t^3-1)$

Strategy hints & answers

(I) Differentiate and simplify:

1. Use product rule with: $y = e^u \cdot \cos(v) \Rightarrow \dfrac{dy}{dx} = e^u \cdot \dfrac{d}{dx}[\cos(v)] + \dfrac{d}{dx}[e^u] \cdot \cos(v)$
   Use chain rule to evaluate the derivatives.
   Answer: $\dfrac{dy}{dx} = -2e^{-2t}\cos(4t) - 4e^{-2t}\sin(4t) \quad = \quad \boxed{-2e^{-2t}[\cos(4t) + 2\sin(4t)]}$

2. Let $f = e^u$. Then $\dfrac{df}{dx} = \dfrac{df}{du} \cdot \dfrac{du}{dx} = e^u \dfrac{du}{dx}$.
   Use product rule with: $u = x \cdot \sin^2(x) = x \cdot v^2$ to find $\frac{du}{dx}$.
   Answer: $\boxed{\dfrac{df}{dx} = e^{x\sin^2 x}[\sin^2(x) + 2x\sin(x)\cos(x)]}$

3. Let $f = u^{1/2}$. Then $\dfrac{df}{dt} = \dfrac{df}{du} \cdot \dfrac{du}{dt} = \dfrac{1}{2}u^{-1/2}\dfrac{du}{dt}$.
   Use quotient rule with: $u = \dfrac{t}{t^2+4}$ to find $\frac{du}{dt}$.
   Answer: $\boxed{\dfrac{df}{dt} = \dfrac{4 - t^2}{2\sqrt{t}\,(t^2+4)^{3/2}}}$

4. Let $g = u^{1/2}$, $u = \sin(v)$, $v = \cos(x)$. Then find $\dfrac{dg}{du}, \dfrac{du}{dv}, \dfrac{dv}{dx}$.
   Multiply and get $\dfrac{dg}{dx} = \dfrac{dg}{du} \cdot \dfrac{du}{dv} \cdot \dfrac{dv}{dx}$
   Answer: $\boxed{\dfrac{dg}{dx} = -\dfrac{\sin(x) \cdot \cos(\cos(x))}{2\sqrt{\sin(\cos(x))}}}$

5. Let $f = e^u$, $u = v^2$, $v = \sin(w)$, $w = 4t + 1$. Find $\dfrac{df}{du}, \dfrac{du}{dv}, \dfrac{dv}{dw}, \dfrac{dw}{dt}$.
   Multiply and get $\dfrac{df}{dt} = \dfrac{df}{du} \cdot \dfrac{du}{dv} \cdot \dfrac{dv}{dw} \cdot \dfrac{dw}{dt}$
   Answer: $\boxed{\dfrac{df}{dt} = 8\, e^{\sin^2(4t+1)}\sin(4t+1)\cos(4t+1)}$

(II) Use the formula: $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

1. $dx/dt = -\sin(t)$, $dy/dt = \sin(t) + t\cos(t)$
   Answer: $\boxed{\dfrac{dy}{dx} = -1 - \dfrac{t\cos(t)}{\sin(t)}}$

2. $dx/d\theta = 1 - 2\sin(\theta)\cos(\theta)$, $dy/d\theta = 2\sec^2(2\theta)$

Answer: $\boxed{\dfrac{dy}{dx} = \dfrac{2\sec^2(2\theta)}{1 - 2\sin(\theta)\cos(\theta)}}$

3. $\dfrac{dx}{dt} = \dfrac{t^2}{(t^3-1)^{2/3}}$, $\dfrac{dy}{dt} = \dfrac{-1-2t^3}{(t^3-1)^2}$

Answer: $\boxed{\dfrac{dy}{dx} = -\dfrac{1+2t^3}{t^2\,(t^3-1)^{4/3}}}$