Quiz: October 22

This is a closed-book quiz, and no team-work or reference materials are permitted.

A government report on housing costs says that the mean price of single-family homes nationwide is \$265,700. We want to see how home prices in Indiana compare with this figure. We collect data on a random sample of 43 homes for sale in Indiana and find a mean price of \$243,300 with standard deviation \$53,600.

Carry out a hypothesis test to determine whether the prices of single-family homes in Indiana are lower than the nationwide figures. Be sure to show all steps, including clear statement of hypotheses, model & conditions, computations and conclusion.

Solution

Let μ = true mean price of single-family homes in Indiana,
I will use a significance level of 10%; i.e., $\alpha = 0.1$.

* Null hypothesis $H_0 : \mu = \$265,700$
  Alt. hypothesis $H_A : \mu < \$265,700$ (1 tail since we want to know if it is lower)

* Check conditions:
  (i) Is the sample independent? Yes, because it was randomly selected and its size is 43, which is less than 10% of all homes in Indiana.
  (ii) Is the sampled data approximately normal? Unable to determine. But with a size of 43, that condition is less critical.
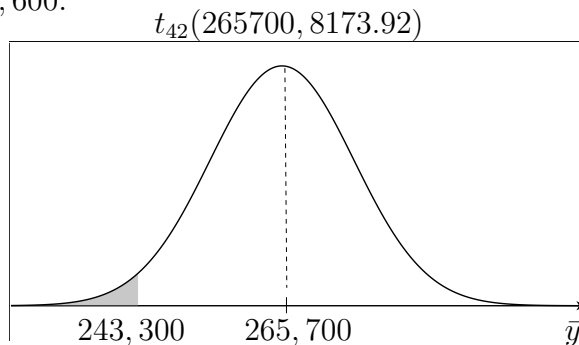
* Sampling distribution model (based on H_0) is: $t_{42}(265,700; \frac{53,600}{\sqrt{43}})$:
  Sample info: $n = 43$, $\bar{y} = \$243,300$, $s = \$53,600$.

  $$\text{SE} = \frac{s}{\sqrt{n}} = \frac{53,600}{\sqrt{43}} = 8173.92$$

  $$t\text{-score} = \frac{\bar{y} - \mu}{\text{SE}} = \frac{243,300 - 265,700}{8173.92}$$
  $$= -2.74$$

  Want to find the area shown in the sketch, which corresponds to $t < -2.74$.

$t_{42}(265700, 8173.92)$

* From t-table, the closest lower df is 40. Our t-score has larger magnitude than the largest one given (i.e., 2.704). Therefore, our P-value is < 0.005 or 0.5%.

* Conclusion: This P-value is below our 10% significance level. Thus, we reject the null hypothesis, and infer that the prices of single-family homes in Indiana are statistically significantly lower than the nationwide average figures.

Grading: Total points possible = 5.

        0.5pt+0.5pt=correct hypotheses + clarify parameter used in them.
        0.5pt+0.5pt=conditions check + compute correct SE.
        1pt=correct df and sampling distribution model $t_{42}(265700, 8173.92)$.
        1pt=compute correct t-score.
        1pt=correct P-value and conclusion.