Quiz: October 15

This is a closed-book quiz, and no team-work or reference materials are permitted.

(1) According to the Central Limit Theorem, what are the mean and the standard deviation of the sampling distribution of sample proportions?

(2) Pew Research polled a random sample of 900 U.S. teens about their Internet use. Of those teens, 540 admitted they had misrepresented their age online to access websites and online services. Construct a 90% confidence interval for the fraction of all U.S. teens who misrepresent their age online to access websites and online services.

Solution

(1) Providing the conditions are met, according to the Central Limit Theorem, the sampling distribution of sample proportions will have:

mean $= p =$ true proportion in the population under consideration

standard deviation $= \sqrt{\frac{p(1-p)}{n}}$, where n is the sample size.

(2) Check conditions for the Central Limit Theorem.

(i) Independence: Reasonable to assume, since sample is random and its size is less than 10% of of all U.S. teens.

(ii) Sample size: There are 540 successes and $900 - 540$ failures, both more than 10. So the sample size is large enough.

Computations:

Confidence interval $= \hat{p} \pm z^* \cdot ME$

Sampled proportion is $\hat{p} = \frac{540}{900} = 0.6.$ Margin of Error $= z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

We want 90% confidence, so: $z* = 1.65.$

Margin of Error $= 1.65 \sqrt{\frac{0.6 \times (1-0.6)}{900}} = 1.65 \times 0.0163 = 0.0269$

Therefore, the confidence interval is: $0.6 \pm 0.0269 = [0.573, 0.627]$

Conclusion: With 90% confidence, the true proprtion of all U.S. teens who will admit they misrepresented their age online to access websites and online services is between 0.573 and 0.627.

Grading: Total points possible $= 5$.

1.5 pt for (1): 0.5pt $=$ correct mean; 1pt $=$ correct SD.

3.5 pt for (2): 0.5+0.5pt $=$ check conditions $+$ correct $\hat{p}$.

1+1pt $=$ compute correct ME $+$ correct C.I.

0.5pt $=$ state a correct conclusion.