Quiz: October 11

This is a closed-book quiz, and no team-work or reference materials are permitted.

(1) According to the Central Limit Theorem, what are the mean and the standard deviation of the sampling distribution of sample means?

(2) A store randomly samples 573 shoppers over the course of a year and finds that 124 of them made their visit because of a coupon they'd received in the mail. Construct a 95% confidence interval for the fraction of all shoppers whose visit was because of a coupon they received in the mail.

Solution

(1) Providing the conditions are met, according to the Central Limit Theorem, the sampling distribution of sample means will have:

mean = true mean of the population under consideration

standard deviation = $\frac{\sigma}{\sqrt{n}}$, where σ is the standard deviation of the population, n is the sample size.

(2) Check conditions for the Central Limit Theorem.

(i) Independence: Reasonable to assume, since sample is random and its size is likely less than 10% of of all shoppers during that year.

(ii) Sample size: There are 124 successes and 449 failures, both more than 10. So the sample size is large enough.

Computations:

Confidence interval = $\hat{p} \pm z^* \cdot ME$

Sampled proportion is $\hat{p} = \frac{124}{537} = 0.2164$. Margin of Error = $z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

We want 95% confidence, so: $z* = 1.96$.

$$\text{Margin of Error} = 1.96\sqrt{\frac{0.2164 \times (1 - 0.2164)}{573}} = 1.96 \times 0.0172 = 0.0337$$

Therefore, the confidence interval is: $0.2164 \pm 0.0337 = [0.183, 0.25]$

Conclusion: With 95% confidence, the true proprtion of all shoppers who visit the store because of a coupon they received in the mail is between 0.183 and 0.25.

Grading: Total points possible = 5.

1.5 pt for (1): 0.5pt = correct mean; 1pt = correct SD.

3.5 pt for (2): 0.5+0.5pt = check conditions + correct $\hat{p}$.
1+1pt = compute correct ME + correct C.I.
0.5pt = state a correct conclusion.