Homework due Oct. 8

Assigned exercises: Ch.7, OpenStax book, pg. 430-438, ex. 22, 23, 24, 25, 31, 32, 33, 56, 57, 63, 64, 67, 81, 82. (14 exercises)
Graded exercises: 31, 32, 33, 64, 67.
Total (maximum) possible points = 20.
3 pt for each of 5 graded problems, plus 5 for completion of the rest.

Exercises from Ch.7, OpenStax book

(31) To apply the CLT, the sample must be large enough, and must be made up of independent observations. Here, the sample size of 400 is likely large enough, but it is not clear whether the sample is random or independent. Assuming the conditions of the CLT are met, the sampling distribution model is $N(3, \frac{0.7}{\sqrt{400}}) = N(3, 0.035)$.

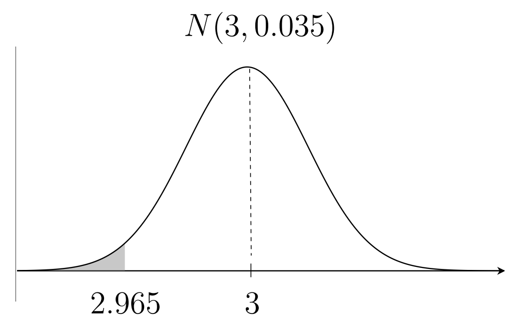
Since $\sum x = 840$ corresponds to a mean of $\bar{x} = \frac{840}{400} = 2.1$, we get $\boxed{z = \frac{2.1-3}{0.035} = -25.71}$

(32) As with Q.31, assuming the conditions are met,
$\sum x = 1186$ corresponds to $\bar{x} = \frac{1186}{400} = 2.965$, and we get $\boxed{z = \frac{2.965-3}{0.035} = -1}$

(33) Since $\sum x = 1186$ corresponds to $\bar{x} = 2.965$, we must find the area in the left tail of the normal distribution $N(3, 0.035)$, as shown in the sketch.

Equivalently, we want the area to the left of $z = -1$ in the standard normal distribution.

Using z-tables, or software, or the 68-95-99.7% rule, the answer is: $\boxed{P(\sum x < 1186) \approx 0.16}$



(64) All parts of this question require the use of the CLT. But not enough information has been provided to determine whether the conditions are met – becasue it is not clear whether the sample is independent, and also the sample size of 49 may not be large enough, if the population is highly skewed or multimodal. If we assume the conditions are met, then we get the following solutions:

(a) According to the CLT: $\boxed{\bar{X} = N(145, \frac{14}{\sqrt{49}}) = N(145, 2) \text{ minutes}}$

(b) z-score for $x = 142$ is: $z = (142 - 145)/2 = -1.5$.
z-score for $x = 146$ is: $z = 0.5$.
Using z-tables or software, the answer is: $\boxed{0.625}$

(c) From z-tables or software, the 80th percentile corresponds to $z = 0.842$.
This gives: $\boxed{145 + 2 \times 0.842 = 146.68 \text{ minutes}}$

(d) (Not graded on HW, but here is the solution.)
The median = 145 minutes = mean, because the distribution is perfectly symmetric.

(67) (a) True, provided the condition of independence is also met. In that situation, the central limit theorem guarantees that the mean of $\bar{X}$ approaches the mean of the population, as the sample size increases.

(b) True, provided the condition of independence is also met. Same reason.

(c) False. When the conditions are met, the CLT says that the standard deviation of $\bar{X} = \dfrac{\text{standard deviation of } X}{\sqrt{n}}$, where n is the sample size.