Homework due Nov. 3

Assigned exercises: OpenIntro supplement, pg.11-12, exercises 4, 5, 6.
Supplemental exercises linked via homework web page:
Pg.673-686, # 1, 7, 11, 13, 15, 26, 30, 35, 37, 39. (13 probs total)
Graded exercises: OpenIntro supplement # 5, and
other supplement, pg.673-686, # 1, 13, 35, 39.
Total (maximum) possible points = 20.
3 pt for each of 5 graded problems, plus 5 for completion of the rest.

Exercises from OpenIntro supplement, pg.11-12

(5) This question is about confidence intervals for mean response values. Thus we must use the standard error expression given by

$$SE = \sqrt{\frac{s_e^2}{n} + SE_{b_1}^2 \times (x^* - \bar{x})^2}$$

The response is given by the regression equation: $\hat{y} = -0.357 + 4.034 \times \text{body wt}$.
Other needed quantities: $s_e = 1.452, n = 144, SE_{b_1} = 0.25, \bar{x} = 2.724, t_{142}^* = 1.977$.
Assuming the conditions are met, the computations and results would be as follows:

(a) For body wt = 5.5 kg: $\hat{y} = -0.357 + 4.034 \times 5.5 = 21.83$ gm.
$SE = \sqrt{\frac{1.452^2}{144} + 0.25^2 \times (5.5 - 2.724)^2} = 0.7045$ gm.
C.I.$= 21.83 \pm 1.977 \cdot 0.7045 = (20.44, 23.22)$ gm.
We are 95% confident that the predicted mean heart weight of cats with a body weight of 5.5 kg lies between 20.44 and 23.22 grams.

(b) and (c) are not graded. Their answers are included in the supplement.

Exercises from supplement linked via homework web page

(1) The conditions that must be satisfied are:

(i) Approximately linear relationship: The scatterplot between the explanatory and response variables suggests this condition is satisfied.

(ii) Approximately normal residuals: The histogram of the residuals is symmetric, and the best we can tell, appears close to normal.

(iii) Constant variability of residuals: While the plot of the residuals exhibits some changes in variability, it is likely within range of acceptable.

(iv) Independent observations: No information is given to help check this condition. One indirect indication may be that the residuals plot shows no obvious trend or pattern.

(13) The administrators are 95% confident that the mean 6-year graduation rate for all top colleges that admit 33% of their applicants is predicted to lie between 87.86 and 89.60 percent.

(35) (a) Let β_1 = slope of the true relationship between weight and fuel efficiency of cars.

Null hypothesis $H_0 : \beta_1 = 0$

Alt hypothesis $H_A : \beta_1 \neq 0$

(b) Conditions: The scatterpplot suggests a linear relationship; the histogram of residuals looks symmetric and close to normal; the residuals plot suggests there may be a slight increase in variability for larger x-values, but it is not too bad; the independence condition may be met, but there is insufficient information to tell.

(c) is not graded, but here is the answer:

(c) From the software output, we have t-score$=\frac{b_1-\beta_1}{SE} = \frac{-8.2136-0}{0.674} = -12.19$, with $df = 48$. The P-value is almost 0. Thus we reject H_0 and infer that there is strong evidence of a linear relationship between weight of a car and fuel efficiency.

(39) (a) This question is about a confidence interval for the mean of the predicted y-values at a specific x-value. Here are the needed numerical quantities and calculations:

$\hat{y} = 48.7393 - 8.2136x = 48.7393 - 8.2136 \times 2.5 = 28.2053$ mpg

$SE = \sqrt{\frac{s^2}{n} + SE_{b_1}^2(x-\bar{x})^2} = \sqrt{\frac{2.413^2}{50} + (0.674)^2(2.5-2.8878)^2} = 0.4298$ mpg

C.I. $= \hat{y} \pm t_{48}^* SE = 28.2053 \pm 2.014 \times 0.4298 = (27.34, 29.07)$ mpg

We are 95% confident that the mean fuel efficiency of all cars weighing 2500 pounds is predicted to lie between 27.34 and 29.07 mpg.

(b) This question is about a prediction interval for y-values at a specific x-value. The key difference from part (a) is in how we calculate the SE. The x-value in this question is also different.

$\hat{y} = 48.7393 - 8.2136x = 48.7393 - 8.2136 \times 3.45 = 20.4024$ mpg

$SE = \sqrt{s^2 + \frac{s^2}{n} + SE_{b_1}^2(x-\bar{x})^2}$

$= \sqrt{2.413^2 + \frac{2.413^2}{50} + (0.674)^2(3.45-2.8878)^2} = 2.4663$ mpg

C.I. $= \hat{y} \pm t_{48}^* SE = 20.4024 \pm 2.014 \times 2.4663 = (15.44, 25.37)$ mpg

We are 95% confident that the fuel efficiency of a car weighing 3450 pounds will lie between 15.44 and 25.37 mpg.