A warmup

Consider the function
$$y = \frac{1}{1 + e^{-(mx+b)}}$$
with x = independent variable, y = dependent variable, m, b = some given numbers.

Let's try to understand this function a bit:

(1) Graph y vs x and see what it looks like. Pick some values for m, b (e.g, $m = 1, b = 0$ would work just fine).

(2) What is the range of y? In fact, go ahead and find: $\lim_{x\to\pm\infty} y$

(3) Show that the function can be rewritten as
$$\frac{y}{1-y} = e^{mx+b} \;\Rightarrow\; \ln\left[\frac{y}{1-y}\right] = mx + b$$

(4) If we define $z = \ln[y/(1-y)]$, this last equation is a straight line relationship between x and z.

(5) One last Q: What is the range of z?

Logistic regression idea

1. We want to construct a regression model to predict the response of a categorical variable y that has two categories (e.g., yes/no, right/wrong).
2. To do this, we first encode the categories as binary numbers.
3. We then find the best linear model for predicting $z = \ln[y/(1-y)]$, of the form
$$\hat{z} = b_0 + b_1 x_1 + b_2 x_2 + \ldots + b_k x_k$$
4. To predict $\hat{y}$ for any set of given inputs, we first find $\hat{z}$, and then compute $\hat{y}$ by inverting the formula $\hat{z} = \ln[\hat{y}/(1-\hat{y})]$.
5. The resulting $\hat{y}$ will be a value between 0-1, and represents a probability corresponding to the categories in y.