an issue, what would be the ideal “shape” of each container? Perform a computer experiment that supports your conclusions based on potential energy considerations.

A Mathematical Model of a Groundwater Contaminant Source

Chlorinated solvents such as trichloroethylene (TCE) are a common cause of environmental contamination at thousands of government and private industry facilities. TCE and other chlorinated organics, collectively referred to as dense nonaqueous phase liquids (DNAPLs), are denser than water and only slightly soluble in water. DNAPLs tend to accumulate as a separate phase below the water table and provide a long-term source of groundwater contamination. A downstream contaminant plume is formed by the process of dissolution of DNAPL into water flowing through the source region as shown in Figure 2.P.2.

In this project, we study a first order differential equation that describes the time dependent rate of dissolved contaminant discharge leaving the source zone and entering the plume. Parameters and variables relevant to formulating a mathematical model of contaminant discharge from the source region are defined in the following table:

- \( A_s \) = cross-sectional area of the source region
- \( v_d \) = Darcy groundwater flow velocity
- \( m(t) \) = total DNAPL mass in source region

![Contaminant plume](image)

**FIGURE 2.P.2** Conceptual model of DNAPL source.


13The output of this model can then be used as input into another mathematical model that in turn describes the processes of advection, adsorption, dispersion, and degradation of contaminant within the plume.

14In porous media flow, the Darcy flow velocity \( v_d \) is defined by \( v_d = Q/A \) where \( A \) is a cross-sectional area available for flow and \( Q \) is the volumetric flow rate (volume/time) through \( A \).
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2. Additional processes due to biotic and abiotic degradation contributing to source decay can be accounted for by adding a decay term to (3) that is proportional to \( m(t) \),

\[
\frac{dm}{dt} = -A_s c_s(t) \left( \frac{m(t)}{\hat{m}} \right)^\gamma - \lambda m, \tag{1}
\]

where \( \lambda \) is the associated decay rate constant. Find solutions of Eq. (1) using the initial condition \( m(0) = m_0 \) for the following cases: (i) \( \gamma = 1 \), (ii) \( \gamma \neq 1 \) and \( \lambda = 0 \), (iii) \( \gamma \neq 1 \) and \( \lambda \neq 0 \). Then find expressions for \( c_s(t) \) using Eq. (2).

Hint: Eq. (1) is a type of nonlinear equation known as a Bernoulli equation. A method for solving Bernoulli equations is discussed in Problem 27 of Section 2.4.

3. Show that when \( \gamma \geq 1 \) the source has an infinite lifetime but if \( 0 < \gamma < 1 \) the source has a finite lifetime. In the latter case, find the time that the DNAPL source mass attains the value zero.

4. Assume the following values for the parameters:

\[
m_0 = 1620 \text{ kg}, \quad c_0 = 100 \text{ mg/l}, \quad A_s = 30 \text{ m}^2, \quad v_s = 20 \text{ m/yr}, \quad \lambda = 0.
\]

Use the solutions obtained in Problem 2 to plot graphs of \( c_s(t) \) for each of the following cases: (i) \( \gamma = 0.5 \) for \( 0 \leq t \leq t_f \) where \( c_s(t_f) = 0 \), (ii) \( \gamma = 2 \) for \( 0 \leq t \leq 100 \) years.

5. Effects of Partial Source Remediation.

(a) Assume that a source remediation process results in a 90% reduction in the initial amount of DNAPL mass in the source region. Repeat Problem 4 with \( m_0 \) and \( c_0 \) in Eq. (2) replaced by \( m_1 = (0.1) m_0 \) and \( c_1 = (0.1) c_0 \), respectively. Compare the graphs of \( c_s(t) \) in this case with the graphs obtained in Problem 4.

(b) Assume that the 90% efficient source remediation process is not applied until \( t_1 = 10 \) years have elapsed following the initial deposition of the contaminant. Under this scenario, plot the graphs of \( c_s(t) \) using the parameters and initial conditions of Problem 4. In this case, use Eq. (2) to compute concentration for \( 0 \leq t < t_1 \). Following remediation, use the initial condition \( m(t_1) = m_1 = 0.1 m(t_1 - 0) = 0.1 m_{\text{lim}} c(t) \) for Eq. (1) and use the following modification of Eq. (2),

\[
\frac{c_s(t)}{c_1} = \left( \frac{m(t)}{0.1 \hat{m}} \right)^\gamma, \quad t > t_1, \tag{5}
\]

where \( c_1 = (0.1)^\gamma c(t_1 - 0) = (0.1)^\gamma \lim_{t \to t_1} c(t) \) to compute concentrations for times \( t > t_1 \). Compare the graphs of \( c_s(t) \) in this case with the graphs obtained in Problems 4 and 5(a). Can you draw any conclusions about the possible effectiveness of source remediation? If so, what are they?