How to Prepare

In 3 words: Practice, practice, practice! There's no substitute.

See Test 2 review for details on how to do this.

Included topics & syllabus

The test will cover the following sections of the textbook:

* Chapter 2: all sections.
* Chapter 3: all sections.
* Chapter 4: all sections except 4.4.
* Chapter 5: sections 5.1-5.3.

Proficiency in Chapter 1 is expected, but no question will directly test this material. Sec. 4.8 and 5.3 were minimally covered. The expectation is mainly: understanding of the evaluation theorem, the "net change" interpretation of definite integrals, and the ability to evaluate antiderivatives and definite integrals of the types seen in class.

Concepts and Definitions:

Chapter 5

- Riemann sum: How do we set up Riemann sums with left/ right/ mid points? What are Δx, x_i, x_i^*?
- How to work with the sigma notation: How do we put expressions into the sigma notation? How do we convert back into expanded notation?
- Area under a curve: How do we represent it in terms of limits of Riemann sums? When do we get an overestimate (an underestimate) of an area? Can some areas be negative?
- The definite integral of $f(x)$ from a to b in terms of Riemann sums? In terms of areas? What is the geometric interpretation of the definite integral? Can a definite integral be negative?
- How to interpret definite integrals as "net change" (analogous to derivatives being "rate of change")?
- Properties of DI's: (a) Interval properties:

  (1) $\int_a^a f(x)\,dx = 0$ (2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

  (3) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

  (b) Comparison properties:

  (1) If $f(x) \geq 0$ for all x in $[a, b]$, then $\int_a^b f(x)\,dx \geq 0$

  (2) If $f(x) \geq g(x)$ for all x in $[a, b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

  (3) If $l \leq f(x) \leq u$ for all x in $[a, b]$, then $l(b-a) \leq \int_a^b f(x)\,dx \leq u(b-a)$
- What is an indefinite integral? an antiderivative? What are the similarities and differences between definite and indefinite integrals? How many antiderivatives does a continuous function have?

Concepts and Defs. (continued):

Chapter 4

- What is an absolute (or global) minimum and maximum of a function?
- What is a local (or relative) minimum and maximum of a function?
- Is a local extremum necessarily an absolute extremum? examples? Is an absolute extremum necessarily a local extremum? examples? Can an endpoint be a local extremum? an absolute extremum?
- Critical points: How do we find all of them?
- How to tell where a function is increasing (decreasing)?
- How to tell where a function is concave up (concave down)?
- What is an inflection point? How do we locate all of them?
- What is an indeterminate form? How do we find limits of indeterminate forms?
- What is the strategy for solving optimization problems?
- What is an antiderivative? How many antiderivatives does x^6 have? How do they differ from each other?

Chapters 2-3

Check previous test review for details. What follows is a short list of only the major concepts:

- Limits of Functions. How to find via geometry & algebra. What are one-sided limits. Limits at infinity. How they relate to Vertical & horizontal asymptotes of $f(x)$.
- Continuity. What does it mean at a point? on the interval $[a, b]$?
- Tangent and secant lines to the graph of a function.
- The concept, and basic definition, of the derivative of a function.
- How to interpret the derivative as "instantaneous rate of change."
- Key rules of differentiation. Implicit differentiation. Logarithmic differentiation.

Key Theorems:

Chapter 5

- The Evaluation Theorem: What does it say? It says:
  Suppose $f(x)$ is continuous on the interval $[a, b]$. Then, the definite integral of $f(x)$ equals the difference of any antiderivative evaluated at the ends of the interval as follows
  $$\int_a^b f(x)\, dx = g(b) - g(a). \qquad \text{Here } g'(x) = f(x).$$

Chapter 4

- Mean Value Theorem: What does it say? To what functions is it applicable?
- L'Hospital's Rule: To what functions is it applicable? How to apply?
- There are also other theoretical results that deal with how to find local & absolute extremes, how to test critical points for type of extreme, etc. Here are some of the key results you should know:
  - First derivative test for local minimum/maximum.
  - 2nd derivative test for local minimum/maximum.
  - Plugin method for absolute extremes of continuous functions on closed interval.
  - If a continuous function only has one critical point (in any type of interval), then any local extreme at that point is also an absolute extreme.

Chapters 2-3

See previous test reviews.

Check out the recitation worksheets for more practice!

Things are better than you think!

Despite the seemingly vast amount of material you must study for this comprehensive final exam, the nature of this course makes the effort much more modest and tractable. For instance, if your preparation for Test 3 was truly thorough and complete, you could almost take the final exam cold, with no further preparation, and still do adequately well on it! Just to be clear, there are a few important topics on the final exam that Test 3 doesn't cover well. These include, for example, sec. 2.3, 2.5, 3.5, 3.7.

Final word

* Test questions will not be designed to be tricky or hard. Instead, my primary goal will be to test your understanding of key concepts, and your ability to apply them for problem solving.

It has been my pleasure, and privilege, to have been your instructor! I wish each of you the very best on the final, as well as in your academic career and beyond!