Quiz 9 - 4/27/2022

Let $y = (x^2 - 1)^3$

(a) Find the intercepts and asymptotes we would expect to see in a graph of y.

(b) Find the intervals of increase/ decrease, and all the extreme values of y.

The intent here is to do everything using calculus and algebra, without the aid of graphing devices. No credit for correct answers – only for correct steps & reasons.

Solution

(a) The given function's domain is $(-\infty, \infty)$.

Since it is a polynomial, it has no vertical or horizontal asymptotes, because $\lim\limits_{x \to a} \neq \pm\infty$ for any finite a, and $\lim\limits_{x \to \pm\infty} \neq L$ for any finite L.

The y-intercept can be found by setting $x = 0$, which gives $y = -1$.

For x-intercepts, $y = 0 \Rightarrow x^2 - 1 = 0$. Thus, x-intercepts are at: $x = \pm 1$.

Answers: | There are no asymptotes. Intercepts: $x = \pm 1$, $y = -1$.

(b) For intervals of increase/ decrease, must determine sign of 1st derivative:

To find critical points of $y = (x^2 - 1)^3$, set $y' = 0$ and solve for x.

$y' = 3(x^2 - 1)^2 \cdot (x^2 - 1)' = 3(x^2 - 1)^2 \cdot 2x$

$\Rightarrow y' = 6x(x^2 - 1)^2$

Therefore, $y' = 0$ when: $x = 0$ and $x = \pm 1$. These are the critical points.

Next, make a sign chart of y':

Since $(x^2 - 1)^2$ is always positive, sign of $y' = $ sign of $6x$.

Therefore, $x < 0 \Rightarrow y' < 0$, and $x > 0 \Rightarrow y' > 0$, as shown below
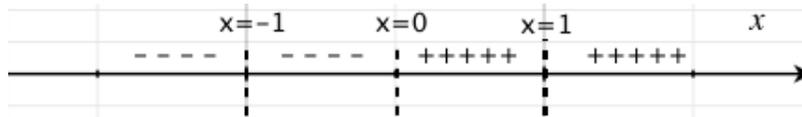
Thus, we know: y is $\downarrow$ on $(-\infty, -1)$ and $(-1, 0)$. y is $\uparrow$ on $(0, 1)$ and $(1, \infty)$.

The sign chart also shows the only local extreme value is at $x = 0$.

Answers: | y is $\downarrow$ on $(-\infty, -1)$ and $(-1, 0)$.
y is $\uparrow$ on $(0, 1)$ and $(1, \infty)$.
There is a local minimum at $(0, -1)$.

Grading: Total points possible $= 6$.

(a) $= 2$ points. (b)$=4$ points.

For (a): 0.5pt $=$ reason why no asymptotes.

1pt $=$ reason why x-intercepts are ± 1.

0.5pt $=$ reason why y-intercept is -1.

For (b): 1pt $=$ get correct y'.

1pt $=$ show how to get correct 3 critical points.

1pt $=$ correct sign analysis to determine intervals of increase/ decrease.

1pt $=$ find/show why local minimum at $(0, -1)$.