Quiz 7 - 4/12/2022

Find dy/dx and simplify: $x^2y^2 + x\sin(y) = 4y$

Solution

Differentiate each term with respect to x.

Term 1:
$$\frac{d}{dx}(x^2y^2) = x^2\frac{d}{dx}y^2 + y^2\frac{d}{dx}x^2 = x^2 \cdot 2y\frac{dy}{dx} + y^2 \cdot 2x$$
$$\therefore \quad \frac{d}{dx}(x^2y^2) = 2x^2y\frac{dy}{dx} + 2xy^2$$
$$\text{Term 2: } \quad \frac{d}{dx}[x\sin(y)] = x\frac{d}{dx}\sin(y) + \sin(y)\frac{d}{dx}x = x \cdot \cos(y)\frac{dy}{dx} + \sin(y) \cdot 1$$
$$\text{Term 3: } \quad \frac{d}{dx}(4y) = 4\frac{dy}{dx}$$

Put everything together and solve for dy/dx:

$$2x^{2}y\frac{dy}{dx} + 2xy^{2} + x\cos(y)\frac{dy}{dx} + \sin(y) = 4\frac{dy}{dx}$$

$$\Rightarrow 2xy^{2} + \sin(y) = 4\frac{dy}{dx} - 2x^{2}y\frac{dy}{dx} - x\cos(y)\frac{dy}{dx}$$

$$= \frac{dy}{dx}[4 - 2x^{2}y - x\cos(y)]$$

$$\therefore \frac{dy}{dx} = \frac{2xy^{2} + \sin(y)}{4 - 2x^{2}y - x\cos(y)}$$

Grading: Total points possible = 6.

2 pt = correctly differentiate term 1.
2 pt = correctly differentiate term 2.
1 pt = correctly differentiate term 3.
1 pt = correct algebra and final result.