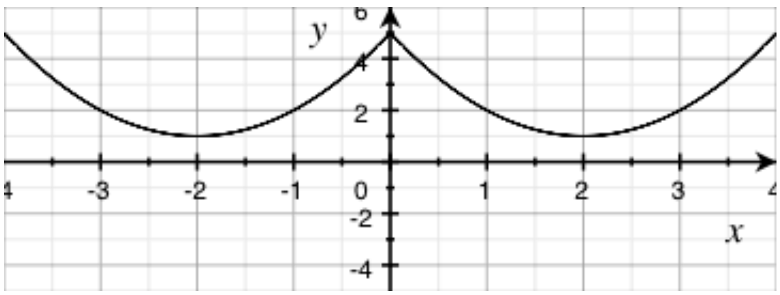
Quiz 5 - 3/15/2022

(I) The graph of some function $y = f(x)$ is shown. Sketch the graph of $f'(x)$. You may do this directly on the graph below. Include a short discussion to justify why your graph of $f'(x)$ is the right solution.



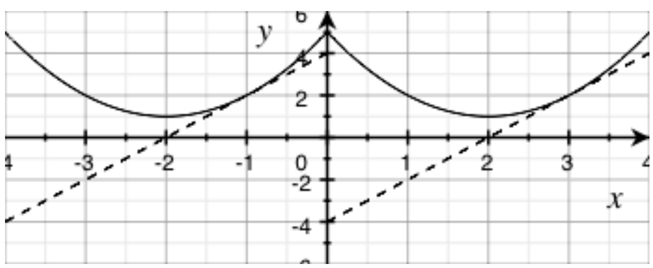
(II) Sketch the graph of a function f that satisfies:

$f'(x) < 0$ for all x, $f''(x) > 0$ if $x < 0$, $f''(x) < 0$ if $x > 0$.

Be sure to identify your axes and include needed labels.

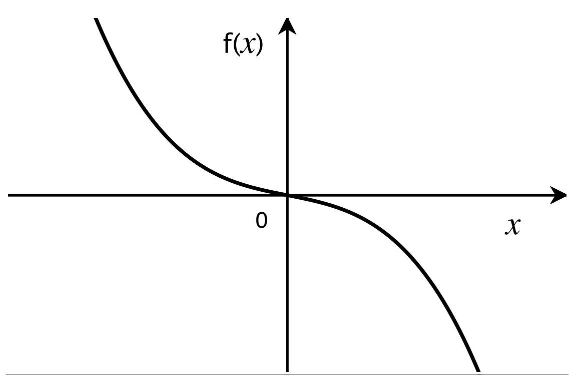
Solution

(I) The dotted curve shown below is the graph of $f'(x)$



Resaoning: f' is 0 wherever the tangent line is horizontal on $f(x)$. There are 2 places where this occurs: $x = -2$ and $x = 2$. Our f' graph intersects the x- axis at those points. When $x = 0$, the kink in the graph of f makes f' undefined. Also note that f is decreasing wherever f' is negative, and f is increasing wherever f' is positive.

(II)



Resaoning: Since we want $f' < 0$ for all x, the graph of f is always decreasing.
When $x < 0$ we want $f''(x) > 0$, which means the graph must be concave up.
When $x > 0$ we want $f''(x) < 0$, meaning a concave down graph.

Grading: Total points possible = 6.

0.5 pt - Any reasonable attempt.

3.5 pt for (I): 1 pt = graph of derivative intersects x-axis at the right places.
1 pt = derivative is undefined at $x = 0$.
1 pt = correct sign throughout.
0.5 pt = explanation.

2 pt for (II): 0.5 pt = any graph of a valid function that is decreasing.
0.5 pt = correct concavity for $x < 0$.
0.5 pt = correct concavity for $x > 0$.
0.5 pt = axis labels.