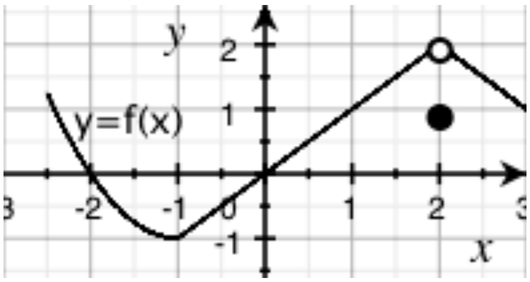
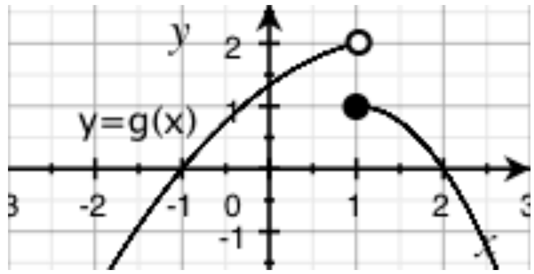
Quiz 3 - 2/22/2022

(I) Evaluate the following limits using the graphs of f and g given below (Note: correct reasoning is far more important here than correct answers):

(a) $\lim_{x\to 1}[f(x)\cdot g(x)]$ (b) $\lim_{x\to 2}[x^2 f(x)]$





(II) Use the mathematical definition of continuity to determine whether the following function is continuous at $x = 0$ (show steps)

$$f(x) = \begin{cases} \dfrac{3x}{x^2 - x} & \text{if } x \neq 0 \\ -3, & \text{if } x = 0 \end{cases}$$

Solution

(I) According to limit laws: Limit of a product = product of the limits, provided both limits individually exist.

(a) When $x \to 1$, we cannot apply the limit laws directly because the limit of $g(x)$ does not exist. However, both functions have left- and right- limits. So, we can apply limit laws to find the limit on each side separately:

$\lim_{x\to 1^-}[f(x)\cdot g(x)] = \lim_{x\to 1^-} f(x)\cdot \lim_{x\to 1^-} g(x) = 1\cdot 2 = 2$

$\lim_{x\to 1^+}[f(x)\cdot g(x)] = \lim_{x\to 1^+} f(x)\cdot \lim_{x\to 1^+} g(x) = 1\cdot 1 = 1$

Since the left- and right- limit are not the same, $\boxed{\lim_{x\to 1}[f(x)\cdot g(x)] = \text{DNE}}$

(b) For this case we can apply limit laws, since both limits exist:

$\lim_{x\to 2}[x^2\cdot f(x)] = \lim_{x\to 2} x^2 \cdot \lim_{x\to 2} f(x) = 4\cdot 2 \boxed{= 8}$

(II) According to the definition, continuity at $x = 0$ requires: $\lim_{x\to 0} f(x) = f(0)$.

In this problem, $f(0) = -3$.

To find $\lim_{x\to 0} f(x)$ we must try to do some algebra and cancel an x

$$\frac{3x}{x^2 - x} = \frac{3x}{x(x-1)} = \frac{3}{x-1} \quad (\text{provided } x \neq 0)$$

Therefore, $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{3}{x-1} = -3$

Since $f(0) = -3$ and $\lim_{x\to 0} f(x) = -3$, it follows that $\underline{f \text{ is continuous at } x = 0.}$

Grading: Total points possible = 6.

3 pt for (I):	1.5pt for (a) + 1.5 pt for (b).
	No credit for correct answers without correct reason.
3 pt for (II):	1 pt = Attempt to apply correct defn of continuity at $x = 0$.
	0.5pt = Find correct $f(0)$.
	1pt = Find correct $\lim_{x\to 0} f(x)$.
	0.5 pt = State correct conclusion.