Quiz 10 - 5/04/2022

(I) Evaluate $\lim\limits_{x\to 1} \dfrac{1 - x + \ln x}{1 + \cos(\pi x)}$. Show all steps.

(II) Setup an optimization function in terms of one unknown variable to solve the following problem:

> "Find the dimensions of a rectangle with perimeter 500 meters whose area is maximum."

You don't need to solve it or find the answer, but must show correct steps leading to the optimization function.

Solution

(I) To find $\lim\limits_{x\to 1} \dfrac{1 - x + \ln x}{1 + \cos(\pi x)}$, first try to plug in $x = 1$ and see if it works.

$\dfrac{1 - 1 + \ln 1}{1 + \cos(\pi)} \sim \dfrac{0}{0}$, which is indeterminate. So, it doesn't work.

Apply L'Hospital's Rule: $\lim\limits_{x\to 1} \dfrac{1 - x + \ln x}{1 + \cos(\pi x)} = \lim\limits_{x\to 1} \dfrac{(1 - x + \ln x)'}{(1 + \cos(\pi x))'} = \lim\limits_{x\to 1} \dfrac{-1 + 1/x}{-\pi \sin(\pi x)}$

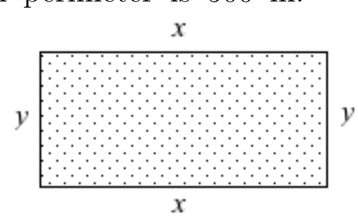
Now plug in $x = 1$ again and check: $\dfrac{-1 + 1}{-\pi \sin(\pi)} \sim \dfrac{0}{0} \Rightarrow$ still indeterminate.

Apply L'Hospital's Rule again: $\lim\limits_{x\to 1} \dfrac{-1 + 1/x}{-\pi \sin(\pi x)} = \lim\limits_{x\to 1} \dfrac{-1/x^2}{-\pi^2 \cos(\pi x)}$

Try to plug in $x = 1$ again: $\dfrac{-1}{-\pi^2(-1)} = -\dfrac{1}{\pi^2}$. It works!

Answer: $\boxed{\lim\limits_{x\to 1} \dfrac{1 - x + \ln x}{1 + \cos(\pi x)} = -\dfrac{1}{\pi^2}}$

(II) Let the two sides of the rectangle be x, y. The given perimeter is 500 m.



$2x + 2y = 500 \quad \Rightarrow \quad y = \dfrac{500 - 2x}{2} = 250 - x.$

The area is: $A = x \cdot y \quad \Rightarrow \quad A = x \cdot (250 - x)$.

The function to be maximized is:

$$\boxed{A(x) = 250x - x^2}$$

Grading: Total points possible = 6.

0.5 pt - Any reasonable attempt.

3.5 pt for (I): 0.5 pt = check whether indeterminate.
1 pt = correctly apply L.H. rule.
1 pt = check indeterminate again, and apply L.H. 2nd time.
1 pt = plug in and get answer.

2 pt for (II): 1.5 pt = show correct steps.
0.5 pt = get correct answer.