Test 2 practice worksheet

1. For each of the following, find $\frac{dy}{dx}$ and simplify:

   (a) $y = \dfrac{3x - 2}{2x + 1}$

   (b) $y = \dfrac{3x - 2}{\sqrt{2x + 1}}$

   (c) $x^2 y + xy^2 = e^{xy}$

   (d) $x = e^{1/t}, \ y = \sin(3t)$

   (e) $y = \sqrt{|1 - 2x|}$

   (f) $y = \sin^2(\cos(2x))$

   (g) $y = x^2 |4 - x^2|$

   (h) $y = e^{\cos^3(5x)}$

   (i) $x^2 \cos(y) + y \sin(2x) = xy$

   (j) $x = \sin(t^2 - \cos t), \ y = \sin^2(3t)$

   (k) $e^{xy} - x^2 y + y^2 x = 5$

   (l) $e^{x/y} - x^2 y + y^2 x = 5$

2. State the limit-based definition of the derivative of a function f. Use your definition to setup an expression for the derivative of each of the following

   (a) $f(x) = (\cos x)^x$

   (b) $f(t) = \sqrt{3t - t^2}$

   (c) $f(x) = \dfrac{g(x)}{h(x)}$

3. Each question below describes a function with certain specific properties. In each case, sketch the graph of a function that has the described properties or, if that is impossible, explain why.

   (a) $f(x)$ has domain $0 \leq x \leq 5$. $f(x)$ is differentiable everywhere, with $f'(x) < 0$. The limit of f does not exist at $x = 2$ and $x = 2.5$.

   (b) $g(x)$ has domain $0 \leq x \leq 5$, and it is continuous everywhere. $g'(x) < 0$ everywhere, except $g'(2) = 0$ and $g'(2.5)$ does not exist.

4. One lesser-known fact about modern, technology-centered warfare, and its myriad variants (e.g., "precision" bombing, "limited" airstrikes, etc.) is its contribution to a dramatic increase in civilian casualties of war, as documented by human rights groups worldwide. As a simple quantitative measure, we can model the percentage of civilian war-deaths as a function of time. Let f denote the % of civilians among those killed in war, as a function of x, the time in years since 1900.

   a. One model for $f(x)$ that approximates the actual known data is given by

   $$f(x) = 9 + e^{0.01x} + 0.2x + 0.0043x^2$$

   What does this model predict for the percent of civilian casualties of war in the year 2019?

   b. Find $f'(x)$ for the $f(x)$ given in a.

   c. Explain the meaning of $f'(2) = 0.23$ in this context, and give its units.

   [For reference, here are best estimates of civilians among the war dead in: WWI=10%, WWII=50%, Vietnam=70%, Iraq=90%]

5. Find y'' (the 2nd derivative of y with respect to x) for each of the following:

    (a) $x = t^4 - t^2$, $\ y = t^3$ (b) $y = \sin(x)e^{x^2}$

    (c) $xy + \sin y = 3$

6. Shown below is the graph of f', the derivative of some function f. Based on this graph, answer the following questions (assume the graph continues to infinity on both ends in the direction shown):

    a. On what interval(s) is f increasing, and on what interval(s) is it decreasing?

    b. At what x-values does f have local minimum and maximum values? Reason?

    c. On what interval(s) is f concave up, and on what interval(s) is it concave down?

    d. Sketch a plausible graph of f.
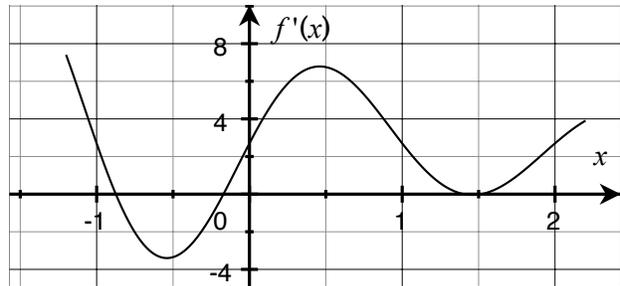
7. According to the theory of relativity, the mass of an object at speed v is given by

$$m(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

    where c is the speed of light and m_0 is the mass when the object is at rest. Find $m'(v)$.

8. A particle moves along the x-axis so that its position at time t is $x = a\sqrt{b^2 + c^2t^2}$, where a, b and c are constants. Find the velocity and acceleration functions of the particle.