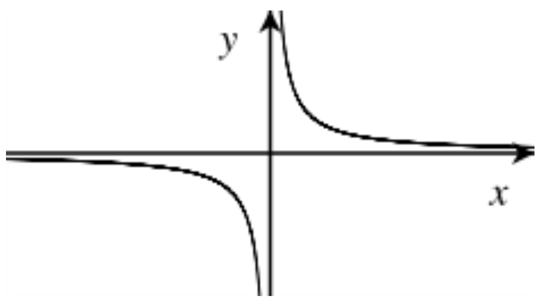
Worksheet 6

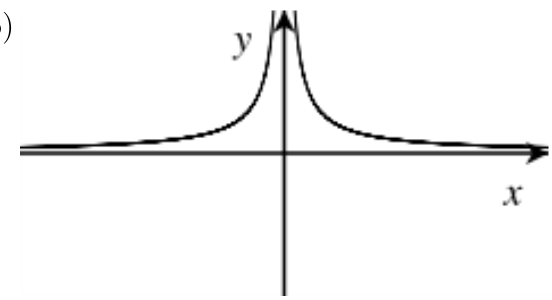
1. Interpret, with correct units and context, the meaning of the derivative in each of the following applications:
   (a) C is the cost in dollars for producing x gallons of a soft drink. What does $C'(x)$ represent? What is the meaning of $C'(10) = 7$?
   (b) A is the amount of a chemical (in mg.) present x minutes after the start of a reaction. What does $A'(x)$ represent? What is the meaning of $A'(3) = -4$?
   (c) F is the fuel consumption (in MPG) of a car when its speed is x miles per hour. What does $F'(x)$ represent? What is the meaning of $F'(14) = 3$?
   (d) R is the rate of gun-related deaths in a state (per 100,000 of the population), when x percent of adults own firearms. What does $R'(x)$ represent? What is the meaning of $R'(14.7) = 6.3$?

2. Shown below are graphs of the derivative $y = f'(x)$ for two different functions. For each case, sketch a qualitatively reasonable graph of $f(x)$.

(a)

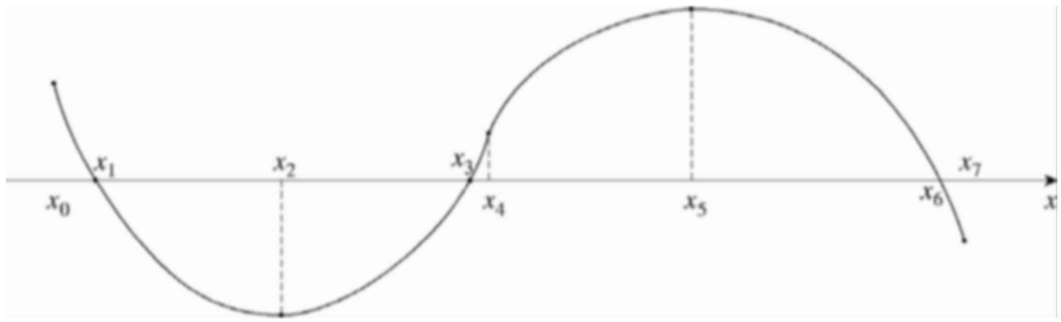


(b)



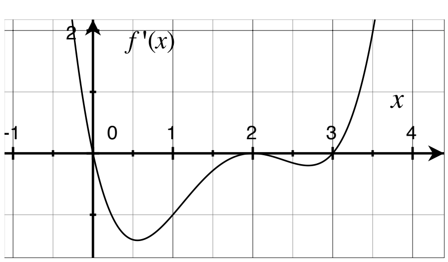
3. Give short answers to each of the following, as instructed.
   (a) Indicate true or false, with reason: "If $f'(a)$ exists, then $\lim_{x \to a} f(x)$ exists."
   (b) Indicate true or false, with reason: "If $\lim_{x \to 3} f(x) = f(3)$, then f is differentiable at $x = 3$."
   (c) Sketch the graph of a function that is increasing at a decreasing rate.
   Also, sketch the graph of one that is decreasing at an increasing rate.
   (d) Suppose $f'(x)$ exists for all x in (a, b). Which of the following statements is guaranteed to be true? (You may pick more than one correct answer.)
      i) $f(x)$ is continuous on (a, b).
      ii) $f(x)$ is defined for all x in (a, b).
      iii) $\lim_{x \to c} f(x)$ exists for all c in (a, b).
      iv) The graph of $f(x)$ is smooth on (a, b). [i.e., it has no kinks or corners]
      v) $f'(x)$ is differentiable on (a, b).

4. The graph of some function $g(x)$ is shown below. Find all the x values where
   (a) g' is increasing. (b) g' is decreasing. (c) $g'' > 0$. (d) $g'' < 0$.

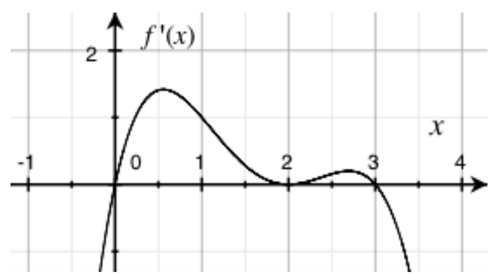


5. Shown below are graphs of the derivative $y = f'(x)$ of four different <u>continuous</u> functions. Based on these graphs, answer the following questions for each function (assume the graphs continue to infinity on both ends in the direction shown):
   i) On what interval(s) is $f(x)$ increasing, and on what interval(s) is it decreasing? Give reasons.
   ii) At what x-values does f have local minimum and maximum values? Reason?
   iii) On what interval(s) is f concave up/down?
   iv) Sketch a qualitatively reasonable graph of $f(x)$, assuming $f(0) = 0$.

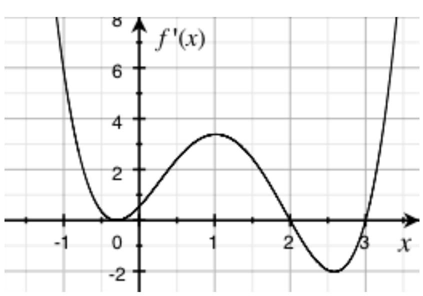
(a)



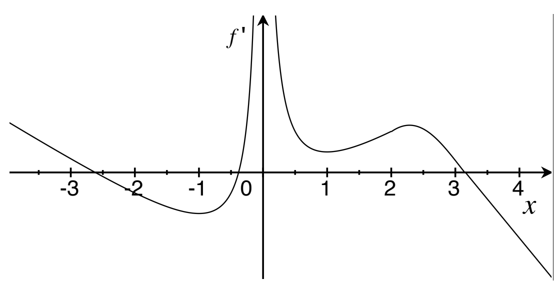
(b)



(c)



(d)



6. (i) In graph (a) of the previous question, is there enough information to tell whether $f(3)$ is less than, or greater than $f(1)$?

   (ii) Similarly, in graph (b) of the previous question, is there enough information to tell whether $f(3)$ is less than, or greater than $f(1)$?

   (iii) In graph (a) of the previous question, is there enough information to tell whether $f(3.5)$ is less than, or greater than $f(1)$?