Worksheet 5

1. The derivative of a function $f(x)$ at a point $x = a$ may be defined using either of the following expressions

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \qquad \text{OR} \qquad f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

Find the derivative of the following functions at the indicated point using each of these expressions

(a) $f(x) = \dfrac{1}{x^2}$ at $x = 3$.

(b) $g(t) = \sqrt{t-1}$ at $t = 5$.

(c) $s(t) = \dfrac{1}{\sqrt{t-1}}$ at $t = a$.

(d) $v(u) = 3u^2 - 1$ at $u = -2$.

(e) $h(x) = 4x + \dfrac{1}{x}$ at $x = a$.

2. The goal of this exercise is to get more comfortable using the above formulas, but without the algebraic clutter. For each function below, setup the two formulas for computing its derivative at an arbitrary point $x = a$. You don't have to find the derivatives – just setup the expressions needed to compute it.

(a) $f(x) = e^{x^2}$

(b) $g(x) = \sin(x^2 + 5)$

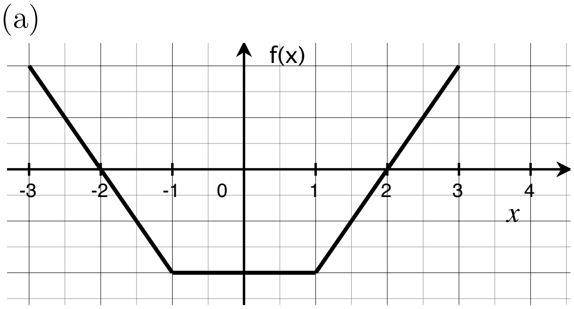
(c) $h(x) = \sqrt{x}\, e^{x^2}$

(d) $f(x) = \dfrac{3x+1}{x-2}$

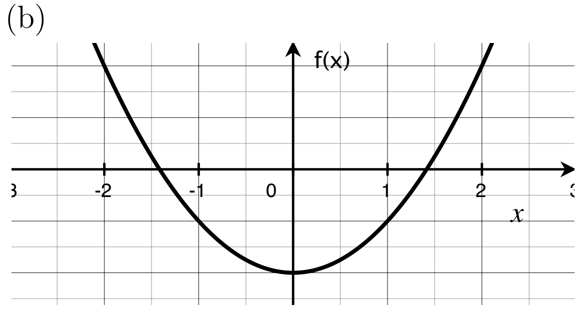
3. Some function $f(x)$ has the following derivative: $f'(40) = -30$.
Interpret, with correct units, what this means in each of the following (different) applications:
   (a) $x =$ price of a product in \$, $f(x) =$ a company's profit in 1000s of \$.
   (b) $x =$ distance from the origin in cm., $f(x) =$ concentration of bacteria in millions per cm.
   (c) $x =$ days after the start of a flu epidemic, $f(x) =$ number of children infected with the flu.

4. Shown below are graphs of various functions $y = f(x)$. Sketch a qualitatively reasonable graph of $f'(x)$ for each function.

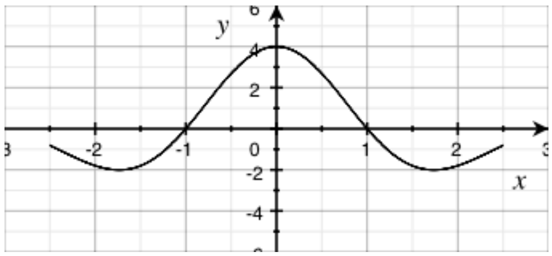
(a)



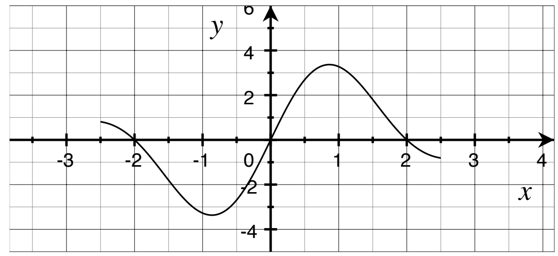
(b)



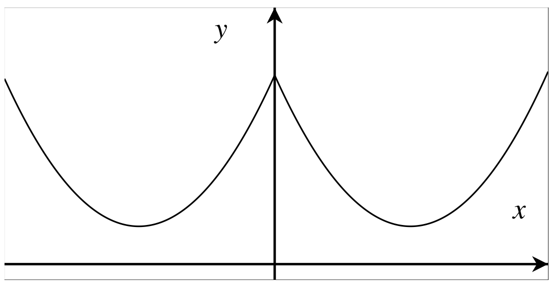
(c)



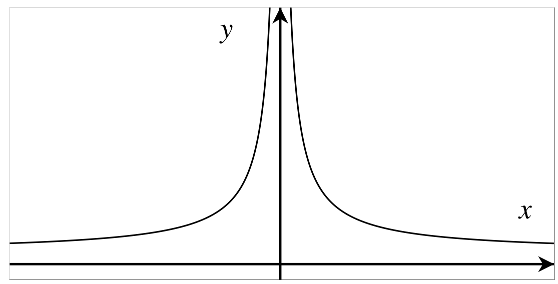
(d)



(e)



(f)



5. Find the equation of the tangent line to the graph of $g(x)$ at $x = -1$, given $g(-1) = 3$ and $g'(-1) = -2$.

6. Suppose the equation of the tangent line to the graph of $y = f(x)$ at $x = 3$ is $y = -4x + 5$. Find $f(3)$ and $f'(3)$.

7. Each limit given below represents the derivative of a function f at $x = a$. Find f and a for each.

   (a) $\lim_{h \to 0} \dfrac{\sqrt{(3+h)^2 - 5} - 2}{h}$

   (b) $\lim_{x \to -4} \dfrac{x^3 + 64}{x + 4}$

   (c) $\lim_{x \to 0} \dfrac{3e^x - 3}{x}$