Worksheet 4

1. Find the indicated limits using algebraic methods:

(a) $\displaystyle\lim_{x\to 3} \frac{|3-x|}{x^2-3x}$

(b) $\displaystyle\lim_{x\to 3} \frac{|x^2-3x|}{3-x}$

(c) $\displaystyle\lim_{x\to 0} \left(\frac{1}{|4x|} - \frac{1}{4x}\right)$

(d) $\displaystyle\lim_{x\to -1} \left(\frac{1}{|4x|} - \frac{1}{4x}\right)$

(e) $\displaystyle\lim_{x\to \infty} \sin(x)$ and $\displaystyle\lim_{x\to \infty} e^{-x}\sin(x)$

(f) $\displaystyle\lim_{x\to -2^-} \frac{x+1}{x+2}$ and $\displaystyle\lim_{x\to -2^+} \frac{x+1}{x+2}$

(g) $\displaystyle\lim_{x\to \infty} (3e^{-x}+1)$ and $\displaystyle\lim_{x\to -\infty} (3e^{-x}+1)$

(h) $\displaystyle\lim_{x\to \infty} \sqrt{\frac{x+8x^2}{2x^2-1}}$

(i) $\displaystyle\lim_{x\to \infty} (\sqrt{x^2+x}-x)$

2. Explain what limits you need to evaluate to find all the vertical and horizontal asymptotes of a function. Evaluate those limits and find the asymptotes of the following:

(a) $f(x) = \dfrac{x^2-x-5}{3x^2+8x-3}$

(b) $g(x) = \dfrac{x^2+5x+6}{3x^2+8x-3}$

(c) $v(x) = \dfrac{3e^x}{e^x-2}$

(d) $h(x) = \dfrac{x^3+5\sin x}{3-7x-2x^2}$

(e) $r(x) = \ln(|x-3|)$

(f) $s(x) = \dfrac{\ln(|x-3|)}{x}$

3. For each piecewise function given below determine where the function is continuous and where it is discontinuous. Justify all claims using the mathematical definition of continuity or by using relevant theorems.

(a) $f(x) = \begin{cases} -x^2, & \text{if } x < -1 \\ 1-x, & \text{if } |x| \le 1 \\ \ln x, & \text{if } x > 1 \end{cases}.$

(b) $f(x) = \begin{cases} e^x, & \text{if } x \le 0 \\ x/2, & \text{if } 0 < x < 4 \\ \sqrt{x}, & \text{if } x \ge 4 \end{cases}.$

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4. Use the Squeeze Theorem to evaluate the following limits

   (a) $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right)$

   (b) $\lim_{x \to 0} x^2 \, e^{\sin\left(\frac{1}{x}\right)}$

   (c) $\lim_{x \to \infty} \frac{\sin x}{x}$

5. Use the Intermediate Value Theorem to check whether the following equations have a root. Be sure to check that the theorem's hypotheses are satisfied before applying it.

   (a) $x^2 + \cos(\pi x) = 4$

   (b) $\frac{x^2+1}{x^2-2} = \frac{1}{2}$

   (c) $\ln(x) = e^{-x}$