Worksheet 3

1. Find the indicated limits using algebraic methods:

(a) $\displaystyle \lim_{x\to 2} \frac{x^2-4}{x^2-x-2}$

(b) $\displaystyle \lim_{x\to 2} \frac{x^4-16}{x^2-x-2}$

(c) $\displaystyle \lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$

(d) $\displaystyle \lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x^2-9}$

(e) $\displaystyle \lim_{x\to 9} \frac{x^2-81}{3-\sqrt{x}}$

(f) $\displaystyle \lim_{x\to -1} \frac{x+1}{2\sqrt{x+2}-\sqrt{x+5}}$

(g) $\displaystyle \lim_{x\to -1} \frac{x^2-3x-4}{2\sqrt{x+2}-\sqrt{x+5}}$

(h) $\displaystyle \lim_{x\to 2} \left(\frac{x^2}{x-2}-\frac{4}{x-2}\right)$

(i) $\displaystyle \lim_{x\to 2} \left(\frac{x^2}{4-x^2}+\frac{1}{x-2}\right)$

(j) $\displaystyle \lim_{x\to 0} \frac{(4+x)^3-64}{x}$

2. The graphs of f and g are given below. Use them to evaluate each of the following limits (give a mathematical justification for your answers):

(a) $\displaystyle \lim_{x\to 1}[f(x)+g(x)]$

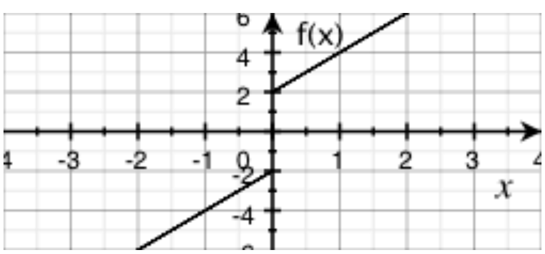
(b) $\displaystyle \lim_{x\to 1}[f(x)\cdot g(x)]$

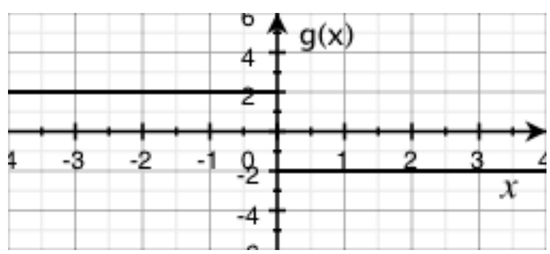
(c) $\displaystyle \lim_{x\to 0}[f(x)+g(x)]$

(d) $\displaystyle \lim_{x\to 0}[f(x)\cdot g(x)]$

(e) $\displaystyle \lim_{x\to 0} \frac{f(x)}{g(x)}$

(f) $\displaystyle \lim_{x\to 0} \frac{f(x)+2}{g(x)}$





3. For each of the following questions, sketch the graph of a function with the indicated properties. Graph must include detailed labels, and indicate open/closed intervals as needed.

(a) $\displaystyle \lim_{x\to -3} g(x) = -2,\ g(-3) = \text{undefined},\ \lim_{x\to 0^+} g(x) = 1,\ \lim_{x\to 0^-} g(x) = -1, g(0) = 0.$

(b) $\displaystyle \lim_{x\to -2^-} f(x) = -1,\ \lim_{x\to -2^+} f(x) = -1,\ \lim_{x\to 1^-} f(x) = 2,\ \lim_{x\to 1^+} f(x) = -2$
$f(-2) = f(1) = 2.$

4. A ball is thrown into the air with a velocity of 40 ft/s. Its height (in feet) t seconds later is given by $y = 40t - 16t^2$.

   (a) Find its average velocity during each of the following time periods:
   (i) $t = 2$ to 2.2, (ii) $t = 2$ to 2.1, and (iii) $t = 2$ to 2.05.

   (b) From these averages, estimate the instantaneous velocity at $t = 2$.

5. In the year 2006 the number of active users on Facebook was about 12 million. By July 2010 the number of active users had grown to over 500 million, according to the company's press release. The following function, based on actual data from Facebook, is a model for the number of active users worldwide (t denotes the number of years after 2006, and N the number of users in millions)

$$N(t) = \frac{12}{0.01 + 0.99\, e^{-0.9t}}$$

   (a) Find the average change in the number of users over the time period beginning with the year 2009 and lasting: 2 years, 1 year, and 0.5 year. In other words, we want to compute three separate averages here.
   (b) From these averages, estimate the slope of the tangent line to the graph of $N(t)$ when the year is 2009. Find the equation of the tangent line.