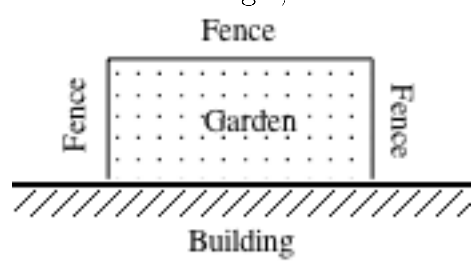
Worksheet 13

1. A fast-food supplier wants to construct open boxes for dine-in orders. The boxes are to be constructed from rectangular pieces of cardboard by cutting out identical squares from each of the four corners, and bending up the sides. If the pieces of cardboard are 10 inches by 12 inches, find the dimensions of the finished box that would maximize the volume. What is the maximum volume?
   [Remember: Make a sketch. Don't forget to prove that your answer is the absolute maximum.]

2. A piece of wire 20 ft long is to be cut into two pieces. One piece is to be bent into a square, and the other into a circle. Determine the length of each piece so that the sum of the areas is maximized.

3. A community school has \$1500 to buy fencing to enclose a rectangular garden along the side of a building. The fence must cover three sides of the rectangle, but no fence is needed along the building (see sketch). The fence along the ends costs \$20 per foot, while the fence along the front costs \$30 per foot. Find the dimensions of the garden (length and width) that would maximize its area. What is the maximum area?



4. Amy is 2 miles offshore in a boat and wishes to reach a coastal town 6 miles down the straight shoreline. She can row 2 miles per hour and can walk 5 miles per hour. Where should she land her boat to reach the town in the least amount of time? How long will it take her to reach the town?

5. A drinking cup is to be made in the shape of a circular cylinder. For a fixed volume, we wish to make the total amount of material used (the circular bottom, plus the cylindrical side) as small as possible. Under this condition, what is the ratio of the height to the diameter?

6. The weekly demand for organic apples at a retail store is given by the function $p = 12e^{-0.01x}$ where p is the price in dollars per pound and x is the number of pounds sold. Find the price they should charge to maximize weekly revenue. What is the maximum revenue?

7. The cost of manufacturing x units of a certain commodity is $c = 600 + 0.04x + 0.002x^2$ dollars. If each units sells for \$10.00, what daily production will maximize profit?

8. Find the following limits:

   (a) $\lim_{x\to 1}\left(\frac{1}{x-1} - \frac{1}{\ln x}\right)$

   (b) $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)$

   (c) $\lim_{x\to 1^+}\left(\frac{x+1}{x-1}\right)$

   (d) $\lim_{x\to 0}\left(\frac{x-\sin x}{x^3}\right)$

9. Find the most general antiderivative of the following functions:

(a) $f(x) = \dfrac{2 + x^3}{x^3}$

(b) $g(x) = \dfrac{1 + 4x}{\sqrt{x}}$

(c) $h(x) = \dfrac{1 + 4x}{x\sqrt{x}}$

(d) $f(x) = \left(1 - \dfrac{1}{\sqrt{x}}\right)\left(1 + \dfrac{1}{\sqrt{x}}\right)$

(e) $g(t) = \sec^2 t + 3\cos t - \dfrac{1}{2\sqrt{t}}$

(f) $h(t) = \pi^3$

(g) $f(x) = \dfrac{2e^x - e^{2x}}{e^x}$

(h) $g(x) = \dfrac{4}{\sqrt{1 - x^2}} + 1$

10. Solve each of the following as instructed:

(a) Show that of all the rectangles with a fixed perimeter, the one with the largest area is a square.

(b) Find the most general form of f if $f''(x) = 12x^2 - 2$.

(c) Find f if $f''(x) = \cos x$, $f'\left(\frac{\pi}{2}\right) = 1$, $f\left(\frac{\pi}{2}\right) = 3$.

(d) True or false:

(i) An antiderivative of a sum of functions $f + g$ is an antiderivative of f plus an antiderivative of g.

(ii) An antiderivative of a product of functions fg is an antiderivative of f times an antiderivative of g.

(e) A cyclist traveling at 40 ft/s decelerates at a constant rate of 4 ft/s^2. How many feet does she travel before coming to a complete stop?

(f) A car is traveling at 50 miles per hour when the brakes are applied, after which it decelerates at a constant rate of 22 ft/s^2. What is the distance covered before the car comes to a complete stop? (For reference, 1 mile = 5280 feet.)