Homework Solutions for Sec. 3.5–3.6

Assigned problems: Sec 3.5: 5, 8, 9, 12, 13, 16, 17, 25, 28, 33.
Sec. 3.6: 3, 5, 9, 10, 17, 18, 23, 25, 27, 33, 35.

Graded problems circled.

Grading scheme: 3 points each for 5 graded problems, plus 10 points for completion of the rest.

Sec. 3.5: Exercise 12: Find $\frac{dy}{dx}$ for $y \cdot \sin(x^2) = x \cdot \sin(y^2)$

Need product rule + chain rule for both terms.

$$* \frac{d}{dx} y \cdot \sin(x^2) = y \cdot \frac{d}{dx}\sin(x^2) + \sin(x^2) \cdot \frac{dy}{dx}$$
$$= y \cdot \cos(x^2) \cdot (2x) + \sin(x^2) \cdot \frac{dy}{dx}$$

$$* \frac{d}{dx} x \cdot \sin(y^2) = x \cdot \frac{d}{dx}\sin(y^2) + \sin(y^2) \cdot \frac{dx}{dx}$$
$$= x \cdot \cos(y^2) \cdot \left(2y\frac{dy}{dx}\right) + \sin(y^2)$$

Put together:

$$2xy \cdot \cos(x^2) + \sin(x^2) \cdot \frac{dy}{dx} = 2xy \cdot \cos(y^2) \cdot \frac{dy}{dx} + \sin(y^2)$$

$$\Rightarrow \frac{dy}{dx}\left[\sin(x^2) - 2xy \cdot \cos(y^2)\right] = \sin(y^2) - 2xy \cdot \cos(x^2)$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{\sin(y^2) - 2xy \cdot \cos(x^2)}{\sin(x^2) - 2xy \cdot \cos(y^2)}} = \frac{2xy \cdot \cos(x^2) - \sin(y^2)}{2xy \cdot \cos(y^2) - \sin(x^2)}$$

Grade: 1 pt = correct derivative of each term = 2 points for 2 terms
1 pt = correctly solve and get dy/dx

Sec. 3.5: Exercise 25: $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$. Find eqn. of tangent at $(0, 1/2)$

Differentiate term-by-term with respect to x.
(Primes denote derivatives with respect to x.)

$$2x + 2y \cdot y' = 2(2x^2 + 2y^2 - x)(2x^2 + 2y^2 - x)'$$

$$\Rightarrow 2x + 2y \cdot y' = 2(2x^2 + 2y^2 - x)(4x + 4y \cdot y' - 1)$$

No need to explicitly solve for y', since we want its value at only one point.

Plug in $(0, 1/2)$ to get:

$$0 + y' = 2\left(0 + \tfrac{1}{2} - 0\right)(0 + 2y' - 1) \Rightarrow y' = 2y' - 1 \Rightarrow \boxed{y' = 1}$$

Slope of tangent at $(0, 1/2)$ is: $y' = 1$

Equation of tangent line: $y = mx + b \Rightarrow y = x + b$
plug in $(0, 1/2)$ and get: $\frac{1}{2} = 0 + b$

$\therefore$ Eqn. of tangent line is: $\boxed{y = x + \frac{1}{2}}$

Grade: 2 pt = correctly differentiate both sides of given eqn.
0.5 pt = plug in $(0, 1/2)$ and get $y' = 1$
0.5 pt = find correct eqn. of tangent line

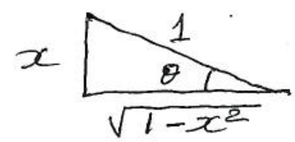
Sec. 3.6: Exercise 10 simplify the expression: $\tan(\sin^{-1} x)$

Let $\sin^{-1} x = \theta$. Then we want to find $\tan(\theta)$.

Since $\sin^{-1} x = \theta$, by defn. of inverse: $x = \sin(\theta)$

The right triangle shows a situation

where $\sin(\theta) = x = \frac{x}{1}$



From this we get the adjacent side $= \sqrt{1-x^2}$

Therefore, $\boxed{\tan(\theta) = \frac{x}{\sqrt{1-x^2}}} = \tan(\sin^{-1} x)$

Grade: Introduce θ and conclude $x = \sin\theta$

0.5 pt → 1.5 pt = construct right Δ and find 3 sides correctly

1 pt = find correct result for $\tan\theta$.

Other methods: Any reasonable trig-based argument after $x = \sin\theta$ is OK too. E.g, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{\sqrt{1-\sin^2\theta}} = \frac{x}{\sqrt{1-x^2}}$

Sec. 3.6: Exercise 23 $y = \cos^{-1}(e^{2x})$

Let $y = \cos^{-1}(u)$, where $u = e^v$ and $v = 2x$

$\frac{dy}{du} = -\frac{1}{\sqrt{1-u^2}}$. $\frac{du}{dv} = e^v$, $\frac{dv}{dx} = 2$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{\sqrt{1-u^2}} \cdot e^v \cdot 2 = \boxed{-\frac{2e^{2x}}{\sqrt{1-e^{4x}}}}$

Grade: 1pt = correct derivative of $\frac{dy}{du}$. 0.5+0.5pt = derivatives of $\frac{du}{dv}$, $\frac{dv}{dx}$. 1pt = put together and get correct answer.

(NOTE: Explicit use of substitution variables not required)

Sec. 3.6: Exercise 35 $f(x) = \sqrt{1-x^2} \cdot \arcsin(x)$

$f'(x) = \left(\sqrt{1-x^2}\right)' \cdot \arcsin(x) + \sqrt{1-x^2} \cdot [\arcsin(x)]'$

$= \frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}} \cdot \arcsin(x) + \sqrt{1-x^2}\left[\frac{1}{\sqrt{1-x^2}}\right]$

$\Rightarrow \boxed{f'(x) = \frac{-x \cdot \arcsin(x)}{\sqrt{1-x^2}} + 1}$

Grade: 0.5pt = attempt product rule at the start

1.5pt = correctly differentiate both terms in P.R.

0.5pt = simplify to a reasonable final answer

1pt