Homework Solutions for Sec. 3.3 – 3.4

Assigned problems: Sec. 3.3: 4, 5, 10, 12, 14, 17, 21, 31, 33, 35, 38(a, b), 39.
Sec. 3.4: 8, 10, 12, 18, 19, 29, 33, 44, 46a, 49, 51, 52, 60, 61, 62, 71, 74, 79, 83.

Graded exercises circled.

Grading scheme: 3 points each for 5 graded problems, plus 10 points for completion of the rest.

Sec. 3.3: Ex. 12

$$y = \frac{1-\sec(x)}{\tan(x)} = \frac{f}{g} \Rightarrow y' = \frac{gf' - fg'}{g^2}$$

$$y' = \frac{\tan(x)\,[1-\sec(x)]' - [1-\sec(x)]\,\tan(x)'}{\tan^2(x)} = \frac{\tan(x)[-\sec(x)\cdot\tan(x)] - [1-\sec(x)]\sec^2 x}{\tan^2(x)}$$

$$= \frac{-\sec(x)\cdot\tan^2(x) - \sec^2(x) + \sec^3(x)}{\tan^2(x)} \quad \text{-------- (Eqn. 1)}$$

$$= \frac{\sec(x)[-\tan^2(x) + \sec^2(x)] - \sec^2(x)}{\tan^2(x)} = \frac{\sec(x) - \sec^2(x)}{\tan^2(x)} = \boxed{\frac{\cos(x) - 1}{\sin^2(x)}}$$

Method 2: Rewrite as $y = \frac{\cos(x) - 1}{\sin(x)}$, because $\sec(x) = \frac{1}{\cos(x)}$, $\tan(x) = \frac{\sin(x)}{\cos(x)}$

$$\Rightarrow y' = \frac{\sin(x)[-\sin(x)] - [\cos(x) - 1]\cos(x)}{\sin^2(x)} = \frac{-\sin^2(x) - \cos^2(x) + \cos(x)}{\sin^2(x)} = \boxed{\frac{-1+\cos(x)}{\sin^2(x)}}$$

Grade: 0.5 pt = correctly plug into Q.R. formula
1 pt = correct derivatives in numerator of Q.R. formula
1 pt = simplify to the form shown in (Eqn. 1)
0.5 pt = simplify further to either of the last 2 forms given above

Sec. 3.3: Ex. 33

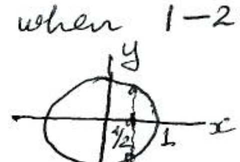
Let $f(x) = x - 2\sin(x)$, $0 \le x \le 2\pi$

f is increasing when $f' > 0$.

$f' = 1 - 2\cos(x) \Rightarrow f' > 0$ when $1 - 2\cos(x) > 0 \Rightarrow 1 > 2\cos(x)$

OR, $\boxed{\frac{1}{2} > \cos(x)}$

From the unit circle,



$\cos(x) < \frac{1}{2}$ when the angle is between $60°$ and $300°$. In radians, $60° = \frac{\pi}{3}$, $300° = \frac{5\pi}{3}$

Therefore, f is increasing when $\boxed{\frac{\pi}{3} < x < \frac{5\pi}{3}}$

Grade: 0.5 pt = Know/reflect understanding that we want $f' > 0$
1 pt = find correct f'
1 pt = show that this leads to the requirement $\cos(x) < \frac{1}{2}$
0.5 pt = Figure out correct answer in radians

Sec 3.4: Exercise 29

$y = \sin(\tan(2x))$

Let $u = \tan(v)$, $v = 2x$. Then $y = \sin(u)$

Then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} \Rightarrow \frac{dy}{du} = \cos(u),\ \frac{du}{dv} = \sec^2(v),\ \frac{dv}{dx} = 2$

$\therefore \frac{dy}{dx} = \cos(u) \cdot \sec^2(v) \cdot 2 = \boxed{2 \cdot \cos(\tan(2x) \cdot \sec^2(2x)}$

Grade: (If using substitutions like u, v, etc)
1 pt = correctly pick sub. variables, e.g. $y = \sin(u)$, $u = \tan(v)$, etc.
1 pt = correctly differentiate y, u, v
1 pt = put everything together correctly and get answer
If NOT using sub. variables, must show key details of in-between steps: E.g. $y' = \cos(\tan(2x)) \cdot (\tan(2x))'$

Sec. 3.4: Exercise 51

Given: $F(x) = f(g(x))$, $f(-2) = 8$, $f'(-2) = 4$, $f'(g(5)) = f'(-2) = 4$

$F'(x) = f'(g(x)) \cdot g'(x) \Rightarrow F'(5) = \underbrace{f'(g(5))}_{=4} \cdot \underbrace{g'(5)}_{=6}$

Since $g(5) = -2$, $f'(g(5)) = f'(-2)$

$\therefore \boxed{F'(5) = 24}$

Grade: 1 pt = get correct derivative of $F'(x)$
0.5 pt = plug in $x = 5$ and get expression for $F'(5)$
1.5 pt = figure out correct numbers to plug in and get answer

Sec. 3.4: Exercise 83

$x = 10 - t^2$, $y = t^3 - 12t$

Slope of tangent $= \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \boxed{\frac{3t^2 - 12}{-2t}}$

For horizontal tangent, $\frac{dy}{dx} = 0 \Rightarrow 3t^2 - 12 = 0 \Rightarrow t^2 = \frac{12}{3}$

$\therefore t = \pm 2$

For vertical tangent, $\frac{dy}{dx} = \infty \Rightarrow -2t = 0 \Rightarrow t = 0$

Points of horizontal tangent: $x_1 = 10 - 4 = 6$, $y_1 = 8 - 24 = -16$

$x_2 = 10 - 4 = 6$, $y_2 = -8 + 24 = 16$

$\Rightarrow$ 2 points are $(6, -16)$, $(6, 16)$

Vertical tangent: $(10, 0)$

Grade: 1 pt = compute correct expression for $\frac{dy}{dx} = \frac{3t^2 - 12}{-2t}$
1 pt = find correct 2 values of t for horizontal tangent, and 1 value of t for vertical
1 pt = find correct (x, y) values for horizontal & vertical cases