Homework Solutions for Sec. 3.1 – 3.2

Assigned exercises: (Sec. 3.1) 10, 11, 15, 16, 18, 19, 20, 26, 30, 42, 47, 48, 52, 68.
Sec. 3.2: 5, 8, 10, 11, 17, 20, 25, 32, 41, 48, 50, 51, 54.

Graded exercises circled.

Grading scheme: 3 points each for 5 graded problems, plus 10 points for completing the rest. Take off 0.5 pt for each missing problem.

Sec. 3.1: Exercise 16

$$y = \sqrt{x}(x-1) = x^{3/2} - x^{1/2}$$

Thus, $y' = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2} \boxed{= \frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}} = \frac{3x-1}{2\sqrt{x}}}$

Method 2: Use product rule.

$$y = x^{1/2}(x-1) \Rightarrow y' = \frac{1}{2}x^{-1/2}(x-1) + x^{1/2} = \frac{1}{2}\sqrt{x} - \frac{1}{2\sqrt{x}} + \sqrt{x}$$
$$= \frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}$$

Grade: 0.5 pt = write out as powers of x (Method 1)
OR, setup correctly in P.R. formula
2 pt = differentiate each term correctly & simplify to $\frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2}$
0.5 pt = rewrite as $\frac{3}{2}x^{1/2} - \frac{1}{2x^{1/2}}$ OR in more simplified form

Sec. 3.1: Exercise 30

Find eqn. of tangent & normal line to $y = (1+2x)^2$ at $(1,9)$

Need to first find slope of tangent by computing derivative at $(1,9)$

$$y = (1+2x)^2 = 1 + 4x + 4x^2 \Rightarrow y' = 4 + 8x. \text{ Plug in } (1,9): \boxed{y' = 12}$$

Eqn. of tangent: $y = 12x + b$. Plug in $(1,9)$ to find b: $9 = 12 + b \Rightarrow b = -3$

Tangent line is: $\boxed{y = 12x - 3}$

Eqn. of normal: $y = -\frac{1}{12}x + b$. The slope of normal is negative reciprocal of tangent slope. Plug in $(1,9)$ to get b: $9 = -\frac{1}{12} + b \Rightarrow b = \frac{109}{12}$

Normal line is: $\boxed{y = -\frac{1}{12}x + \frac{109}{12}}$

Grade: 1 pt = differentiate correctly and get $y' = 12$
1 pt = plug in needed stuff and get correct eqn. of tangent line
1 pt = Know slope of Normal line and find its intercept.

Sec 3.1: Exercise 48

$f(x) = x^3 - 4x^2 + 5x$

Want to find intervals on which f is concave up.

Strategy: Find f'' and look for intervals where it is positive.

$$f' = 3x^2 - 8x + 5 \Rightarrow \boxed{f'' = 6x - 8}$$

$$f'' > 0 \Rightarrow 2(3x-4) > 0 \Rightarrow 3x - 4 > 0 \Rightarrow \boxed{x > \frac{4}{3}}$$

Answer: f is concave up when $x > \frac{4}{3}$, OR on the interval $(4/3, \infty)$

Grade: 1 pt = find correct f' ; 1 pt = get correct f''
1 pt = determine interval where $f''>0$

Sec. 3.2: Exercise 17

$$y = \frac{v^3 - 2v\sqrt{v}}{v} \text{ : Differentiate with respect to } v.$$

Method 1: Simplify and apply power rule.

$$y = v^2 - 2\sqrt{v} = v^2 - 2v^{1/2}$$

$$\frac{dy}{dv} = 2v - 2\left(\tfrac{1}{2} v^{-1/2}\right) = 2v - v^{-1/2} = \boxed{2v - \frac{1}{\sqrt{v}}}$$

Method 2: Apply quotient rule and simplify the mess.
Details left to the reader!

Grade: Method 1: 1 pt = simplify to $v^2 - 2v^{1/2}$
2 pt = differentiate each term correctly
Method 2: 0.5 pt = setup correctly in QR formula
1.5 pt = correct derivatives in numerator of Q.R.
1 pt = simplify the mess

Sec. 3.2: Exercise 50

(a) $f(20) = 10{,}000$ means: When the selling price is \$20 per yard, the quantity of fabric sold is 10,000 yards.

$f'(20) = -350$ means: When the selling price is \$20 per yard, the quantity sold is decreasing with respect to price at the rate of 350 yards per dollar.

(b) $R(p) = p \cdot f(p) \Rightarrow R'(p) = p f'(p) + f(p)[p]'$

$$= p f'(p) + f(p)$$

$$R'(20) = 20 \cdot f'(20) + f(20) = -7000 + 10{,}000 = 3000$$

$R'(20) = 3000$ means: When the selling price is \$20 per yard, the revenue is increasing with respect to price at the rate of \$3000 per dollar.

Grade: (a) = 1.5 point. (b) = 1.5 points
For (a): 0.5 pt = correctly interpret $f(20)$
1 pt = correctly interpret $f'(20)$
For (b): 1 pt = get correct expression for $R'(p)$ and evaluate $R'(20)$
0.5 pt = write correct interpretation of $R'(20)$

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