Homework Solutions for Sec. 2.7–2.8

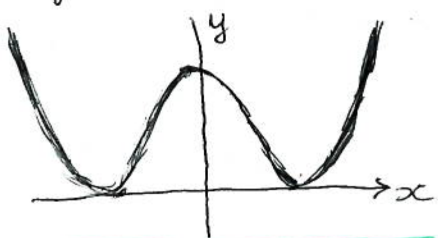
Assigned exercises: Sec. 2.7: 3, 4, 8, 9, 10, 19, 24, 28, 33a, 34a, 36, 41.
Sec. 2.8: 3, 4, 5, 6, 8, 9, 12, 16, 18, 25, 26.

Graded exercises circled.

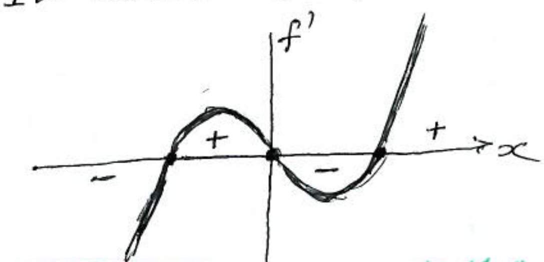
Grading Scheme: 3 points each for 5 graded problems, plus 10 points for completing the rest. Deduct 0.5 pt for each missing exercise.

Sec. 2.7: Exercise 4

Given graph: $y = f(x)$



Its derivative f'



Grade: 1.5 pt = if f' graph touches or crosses x-axes at the 3 points shown; 1.5 pt = the graph of f' is positive & negative in the right places. Partial credit if some of it right.

Sec. 2.7: Exercise 24

Find derivative of $f(x) = x + \sqrt{x}$ using the definition.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h+\sqrt{x+h}) - (x+\sqrt{x})}{h}$$

$$= \lim_{h\to 0} \frac{h + \sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}} + \lim_{h\to 0} \frac{\sqrt{x+x} - \sqrt{x}}{h} = 1 + \underbrace{\lim_{h\to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}}_{\text{Rationalized}}$$

$$= 1 + \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \boxed{= 1 + \frac{1}{2\sqrt{x}}}$$

Method 2: Using other form of definition.

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} = \frac{(x + \sqrt{x}) - (a + \sqrt{a})}{(x-a)} = \lim_{x\to a} \frac{(x-a) + (\sqrt{x} - \sqrt{a})}{x - a}$$

$$= \lim_{x\to a} \frac{(\cancel{x-a})}{(\cancel{x-a})} + \lim_{x\to a} \left[\frac{\sqrt{x} - \sqrt{a}}{x - a}\right] = 1 + \lim_{x\to a} \frac{\cancel{x-a}}{(\cancel{x-a})(\sqrt{x}+\sqrt{a})}$$

$$\Rightarrow \boxed{f'(a) = 1 + \frac{1}{2\sqrt{a}}}$$

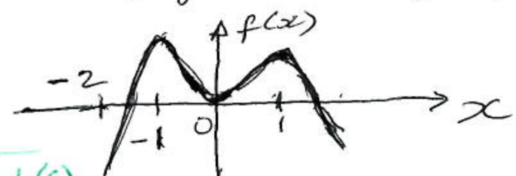
Grade: 0.5 pt = plug $f(x)$ correctly into one of the two definitions
1 pt = split the limit into 2 terms, and find the 1st term reduces to 1.
1.5 pt = find correct limit of 2nd term via rationalizing

Sec. 2.8: Exercise 4

(a) f is ↑ on: $(-2, -1)$ and $(0, 1)$
f is ↓ on: $(-1, 0)$ and $(1, 2)$
} Reason: when $f' < 0$, f increases and $f' < 0$, f decreases

(b) f has local maximum at: $x=-1$ and $x=1$
f has local minimum at: $x=0$
Reason: Local maxima occur when f' goes from positive to negative
Local minima happen when f' goes from negative to positive

(c) One possible graph of $f(x)$ is shown in the sketch.



Grade: 1 point each for (a), (b) and (c)
For (a): 0.5 pt each for correct ↑ and ↓ intervals
For (b): 0.5 pt each for correct points of minimum & maximum
For (c): As long as graph turns around at the right x-locations, with correct intervals of increase/decrease, it is sufficient.
Reasons not required for (a), (b) - but do complain if it is missing!

Sec. 2.8: Exercise 12

$S(t)$ = position, $S'(t)$ = Velocity, $S''(t)$ = acceleration

(a) particle is moving right when: $0<t<2$ and $4<t<6$ (or ∞)
OR, intervals $(0,2)$ and $(4,6)$

Reason: when S ↑, it moves right, when S ↓ it moves left.
Particle is moving left when: $2<t<4 \Rightarrow (2,4)$

(b) Particle has + acceleration when: $3<t<6$ (or ∞) $\Rightarrow (3,6)$
has − acceleration when: $0<t<3 \Rightarrow (0,3)$

Grade: (a) = (b) = 1.5 points
For (a): 1 pt = correct move to right; 0.5 pt = correct left move
For (b): 50/50 split between + and − acceleration
Reasons not required for full credit; but complain if missing

Sec. 2.8: Exercise 25 $f'(x) = x \cdot e^{-x^2}$

(a) Since e^{-x^2} is always positive, the sign of f' depends solely on the sign of x. Thus, $f'>0$, if $x>0$, and $f'<0$, if $x<0$.
$\therefore$ f is decreasing on $(-\infty,0)$, and increasing on $(0,\infty)$.

(b) f' does not have a maximum value, since f' never goes from + to −
f' does have a minimum value at $x=0$, because f' goes from − to +.

Grade: (a) = (b) = 1.5 points each.
For each: 0.5 pt = correct answers; 1 pt = reasons