Homework Solutions for Sec. 2.6

Assigned exercises: 3a, 6, 10a, 12, 14, 15, 19, 22, 25, 28, 32, 41, 43, 47, 50

Graded exercises circled.

Grading Scheme: 3 points each for 5 graded exercises, plus 10 points for completion of the rest.

Exercise 10a: Find slope of tangent to $y = 1/\sqrt{x}$ at $x=a$.

Slope of tangent at $x=a$ is the derivative at $x=a$. Can use either of 2 formulas that define the derivative.

$$y' = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{1/\sqrt{x} - 1/\sqrt{a}}{x-a} = \underbrace{\lim_{x\to a} \frac{\sqrt{a}-\sqrt{x}}{\sqrt{x}\sqrt{a}(x-a)}}_{\text{By taking common denom. of numerator}}$$

$$= \lim_{x\to a} \frac{(\sqrt{a}-\sqrt{x})(\sqrt{a}+\sqrt{x})}{\sqrt{x}\cdot\sqrt{a}(x-a)(\sqrt{a}+\sqrt{x})} \quad \text{(by rationalizing)}$$

$$= \lim_{x\to a} \frac{(a-x)}{\sqrt{x}\cdot\sqrt{a}(x-a)(\sqrt{a}+\sqrt{x})} = \lim_{x\to a} \frac{-1}{\sqrt{x}\cdot\sqrt{a}(\sqrt{a}+\sqrt{x})}$$

Plug in $x=a$: $\lim_{x\to a} \frac{-1}{\sqrt{x}\sqrt{a}(\sqrt{a}+\sqrt{x})} = -\frac{1}{a(2\sqrt{a})} = \boxed{-\frac{1}{a^{3/2}}}$

slope of tangent at $x=a$

Grade: 0.5 pt = correctly plug $f(a)$ into one of 2 formulas for f'
1.5 pt = do the needed algebra correctly (common denom + rationalize)
0.5 pt = plug in $x=a$ and get answer

Exercise 15: Equation of motion is: $s(t) = 1/t^2$

Velocity = derivative of $s(t)$ at $t=a$. Find it by applying either 2 formulas

$$V(a) = \lim_{h\to 0} \frac{s(a+h)-s(a)}{h} = \frac{1/(a+h)^2 - 1/a^2}{h} = \frac{a^2-(a+h)^2}{a^2(a+h)^2 h}$$

$$= \lim_{h\to 0} \frac{\cancel{a^2} - (\cancel{a^2}+2ah+h^2)}{a^2(a+h)^2 h} = \lim_{h\to 0} \frac{-\cancel{h}(2a+h)}{a^2(a+h)^2 \cancel{h}}$$

$$\therefore V(a) = \lim_{h\to 0} \frac{-(2a+h)}{a^2(a+h)^2} = \frac{-2a}{a^2(a)^2} = \boxed{-\frac{2}{a^3}}$$

$V(a) = -\frac{2}{a^3}$. Therefore, $V(1) = -2$ m/s; $V(2) = -\frac{1}{4}$ m/s; $V(3) = -\frac{2}{27}$ m/s

Grade: 0.5 pt = correctly plug in $s(t)$ into derivative formula
1 pt = do the needed algebra
0.5 pt = plug in $h=0$ (or $t=a$) and get $V(a) = -\frac{2}{a^3}$
1 pt = plug into $V(a)$ formula & evaluate at $t=1$ and $t=2$

Exercise 19: Given curve is $y = f(x)$. Given eqn. of tangent line at $x=2$ is $y = 4x-5$.

Since slope of the tangent line is 4, it follows that $\boxed{f'(2)=4}$

The value of $f(2)$ can be found by plugging in $x=2$ in the tangent line equation: $\boxed{f(2) = 4(2)-5 = 3}$

Grade: 1.5 pt = find correct $f'(2)$

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Take off 1 point if only answers given. Otherwise, any steps sufficient

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Exercise 28: Find $f'(a)$ for $f(t) = 2t^3 + t$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \frac{2(a+h)^3 + (a+h) - 2a^3 - a}{h}$$

$$= \lim_{h\to 0} \frac{2(\cancel{a^3} + 3a^2h + 3ah^2 + h^3) + \cancel{a} + h - \cancel{2a^3} - \cancel{a}}{h}$$

$$= \lim_{h\to 0} \frac{6a^2h + 6ah^2 + 2h^3 + h}{h} = \lim_{h\to 0} \frac{\cancel{h}(6a^2 + 6ah + 2h^2 + 1)}{\cancel{h}}$$

Plug in $h=0 \Rightarrow$ $= 6a^2 + 0 + 0 + 1 = 6a^2 + 1$

$\therefore \boxed{f'(a) = 6a^2+1}$

Grade: Similar to Exercise 10a above

Exercise 50:

Quantity of coffee sold, in pounds $= f(p)$
where p = price in dollars per pound.

(a) $f'(8)$ is the instantaneous rate of change in the quantity of coffee sold, with respect to price, when the price is \$8.

Its units are: $\frac{\text{pounds}}{\text{dollar per pound}}$ OR $\frac{\text{pounds}}{\text{dollar}}$ OR pounds per dollar

Although these are not 100% equivalent to the 1st answer, they are reasonable

(b) From basic economic intuition, we know that quantity sold decreases as price increases. Thus we expect $f'(8)$ to be negative.

Grade: Only (a) is graded.

2.5 pts for interpretation - Key items it must include is the idea of rate of change with respect to price and the understanding that it is "when $p = \$8$"

0.5 pt = correct units of $f'(8)$