Homework Solutions for Sec. 2.5

Assigned excercises: 4, 7, 8, 20, 24, 27, 33, 34, 37, 40, 42, 47, 52, 53, 54.

Graded problems circled.

Grading scheme: 3 points for each of 5 graded problems, plus 10 points for completing the rest. Take off 1 point for each missing problem.

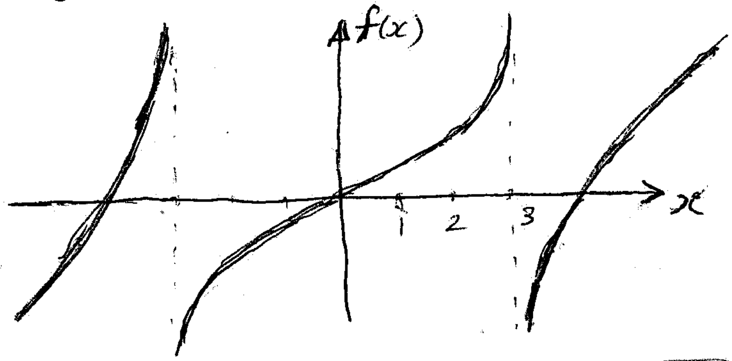
Exercise 8:

f is odd

$\lim_{x \to 3^+} f(x) = -\infty$

$\lim_{x \to 3^-} f(x) = \infty$

$\lim_{x \to \infty} f(x) = \infty$



Grade: 1.5 pt = get correct graph on positive side of x-axis
1 pt = correctly reflect about the origin & get an odd function
0.5 pt = axis labels

Exercise 24

$$\lim_{t \to -\infty} \frac{t^2+2}{t^3+t^2-1}$$

Divide by t^3: $\dfrac{(t^2+2)/t^3}{(t^3+t^2-1)/t^3} = \dfrac{1/t + 2/t^3}{1 + 1/t - 1/t^3}$

Limit as t goes to $-\infty$: $\dfrac{\lim_{t \to -\infty} (1/t + 2/t^3)}{\lim_{t \to -\infty} (1 + 1/t - 1/t^3)} = \dfrac{0+0}{1+0-0} = \dfrac{0}{1} = 0$

because all terms of the form c/t^n go to 0 as $t \to -\infty$.

Therefore, $\boxed{\lim_{t \to -\infty} \dfrac{t^2+2}{t^3+t^2-1} = 0}$

Grade: 1 pt = attempt to divide every term by t^3
1 pt = do it correctly
1 pt = set appropriate terms to 0 as $t \to -\infty$, and get answer

Exercise 27

$$\lim_{x \to \infty} \left[\sqrt{9x^2+x} - 3x\right]$$

Rationalize first: $\dfrac{(\sqrt{9x^2+x} - 3x)(\sqrt{9x^2+x} + 3x)}{\sqrt{9x^2+x} + 3x} = \dfrac{9x^2 + x - 9x^2}{\sqrt{9x^2+x} + 3x}$

Divide numerator & denom by x: $\dfrac{x/x}{(\sqrt{9x^2+x} + 3x)/x} = \dfrac{1}{\sqrt{9 + 1/x} + 3}$

Take limit as $x \to \infty$: $\dfrac{\lim_{x \to \infty} (1)}{\lim_{x \to \infty} \left[\sqrt{9 + 1/x} + 3\right]} = \dfrac{1}{\sqrt{9+0} + 3} \boxed{= \dfrac{1}{6}}$ Answer

Grade: 1 pt = attempt to rationalize with correct conjugate $\sqrt{9x^2+x} + 3x$
1 pt = do the algebra correctly
1 pt = correctly divide result by x and take limit

Exercise 40

Find the asymptotes of $y = \dfrac{x^2+1}{2x^2-3x-2}$

For vertical: $\lim_{x\to a} y = \infty$ or $-\infty$ implies $x = a$ is vertical asymptote

y goes to ∞ at points where denominator $= 0$.

$$2x^2-3x-2=0 \Rightarrow (2x+1)(x-2)=0 \Rightarrow x=2,\ x=-1/2$$

check numerator: $2^2+1 \neq 0$ and $(-1/2)^2+1 \neq 0$

$\therefore$ Vertical asymptotes are at $\boxed{x=2 \text{ and } x=-1/2}$

Horizontal: $\lim_{x\to\pm\infty} y = \lim_{x\to\pm\infty} \dfrac{x^2+1}{2x^2-3x-2} = \lim_{x\to\pm\infty} \dfrac{1+1/x^2}{2-3/x-2/x^2}$

$$\lim_{x\to\pm\infty} \frac{1+1/x^2}{2-3/x-2/x^2} = \frac{\lim_{x\to\pm\infty}(1+1/x^2)}{\lim_{x\to\pm\infty}\left(2-\frac{3}{x}-\frac{2}{x^2}\right)} = \frac{1+0}{2-0-0}$$

$\therefore$ Horizontal asymptote is at $\boxed{y = 1/2}$

Grade: 0.5 pt = Factor denom correctly for V.A.

1 pt = correctly solve and get 2 vertical asymptotes

0.5 pt = attempt to take limit as $x \to \infty$

1 pt = do it correctly, with steps — answer alone not enough

Exercise 53.

Find $\lim_{x\to\infty} f(x)$, given that $\dfrac{10e^x-21}{2e^x} < f(x) < \dfrac{5\sqrt{x}}{\sqrt{x-1}}$

Can try using the squeeze theorem.

$$\lim_{x\to\infty} \frac{10e^x-21}{2e^x} = \lim_{x\to\infty} \frac{10-21e^{-x}}{2} = \frac{10-0}{2} = 5$$

$$\lim_{x\to\infty} \frac{5\sqrt{x}}{\sqrt{x-1}} = \lim_{x\to\infty} \frac{5}{\sqrt{1-1/x}} = \frac{5}{\sqrt{1-0}} = 5$$

By the squeeze theorem, since $\lim_{x\to\infty} \dfrac{10e^x-21}{2e^x} = 5 = \lim_{x\to\infty} \dfrac{5\sqrt{x}}{\sqrt{x-1}}$

it follows that $\boxed{\lim_{x\to\infty} f(x) = 5}$

Grade: 0.5 pt = attempt to apply squeeze theorem

1 pt = get correct $\lim_{x\to\infty} \dfrac{10e^x-21}{2e^x}$, with at least some minimal steps

1 pt = get correct $\lim_{x\to\infty} \dfrac{5\sqrt{x}}{\sqrt{x-1}}$, with at least some steps

0.5 pt = conclude/state that $\lim_{x\to\infty} f(x) = 5$