Homework Solutions for Sec. 2.3 – 2.4

Assigned exercises: Sec. 2.3: 2, 8, 11, 13, 16, 17, 18, 22, 29, 31, 34, 37, 48.
Sec. 2.4: 4, 9, 14, 15 28, 31, 34, 35, 38, 41, 45a.

Graded problems circled.

Grading scheme: 3 points for each of 5 graded problems, plus 10 points for completing the rest. Take off 0.5 pt for any missing problem.

Sec. 2.3: Exercise 13

$\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$. Try plug in first: $\dfrac{9-9}{18-21+3} = \dfrac{0}{0}$ ☹ Doesn't work!

Try factor and cancel: $\dfrac{t^2-9}{2t^2+7t+3} = \dfrac{\cancel{(t+3)}(t-3)}{\cancel{(t+3)}(2t+1)} = \dfrac{t-3}{2t+1}$, if $t \neq -3$

$\lim_{t\to -3}\begin{pmatrix}\text{original}\\ \text{stuff}\end{pmatrix} = \lim_{t\to -3}\left[\dfrac{t-3}{2t+1}\right] = \dfrac{-3-3}{-6+1} = \dfrac{-6}{-5} = \boxed{\dfrac{6}{5}}$

Grade: 0.5 pt = any attempt to factor and cancel
1.5 pt = do it correctly and get to $\dfrac{t-3}{2t+1}$
1 pt = correctly plug in -3 and get answer.
($-1/2$ pt if sign error in final answer)

Exercise 18

$\lim_{h\to 0} \dfrac{\sqrt{1+h}-1}{h}$. Try plug in first: $\dfrac{\sqrt{1+0}-1}{0} = \dfrac{0}{0}$ ☹

Rationalize: Multiply & divide by conjugate $= \sqrt{1+h}+1$

$\dfrac{(\sqrt{1+h}-1)(\sqrt{1+h}+1)}{h(\sqrt{1+h}+1)} = \dfrac{1+h-1}{h(\sqrt{1+h}+1)} = \dfrac{\cancel{h}}{\cancel{h}(\sqrt{1+h}+1)} = \dfrac{1}{\sqrt{1+h}+1}$, if $h \neq 0$

$\lim_{h\to 0}\begin{pmatrix}\text{original}\\ \text{stuff}\end{pmatrix} = \lim_{h\to 0} \dfrac{1}{\sqrt{1+h}+1} = \boxed{\dfrac{1}{2}}$

Grade: 0.5 pt = attempt to rationalize, even if incorrect
2 pt = do it correctly, and simplify to $\dfrac{1}{\sqrt{1+h}+1}$
0.5 pt = plug in $h=0$ and get answer

Exercise 34:

$\lim_{x\to -6} \dfrac{2x+12}{|x+6|}$. Because of abs value, look for left/right limits

Left: $\lim_{x\to -6^-} \dfrac{2x+12}{|x+6|} = \lim_{x\to -6^-} \dfrac{2\cancel{(x+6)}}{-\cancel{(x+6)}} = -2$

Right: $\lim_{x\to -6^+} \dfrac{2x+12}{|x+6|} = \lim_{x\to -6^+} \dfrac{2\cancel{(x+6)}}{\cancel{x+6}} = 2$

Since left limit $\neq$ right limit, the limit $x \to -6$ does not exist.

$\lim_{x\to -6}\begin{pmatrix}\text{original}\\ \text{stuff}\end{pmatrix} = \text{DNE}$

Grade: 0.5 pt = attempt to find left/right limits
2 pt = get correct left and right limit.
0.5 pt = state correct conclusion, i.e., limit DNE

Sec. 2.4
Exercise 15

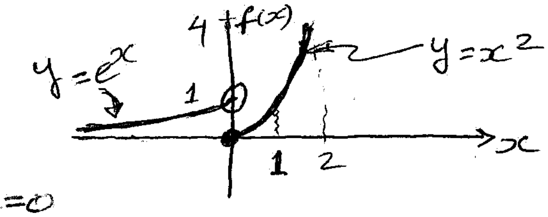
$$f(x)=\begin{cases} e^x, & \text{if } x<0 \\ x^2, & \text{if } x\geq 0 \end{cases}, \quad a=0$$

If f is continuous at $x=a$, then we must have $\lim_{x\to a} f(x) = f(a)$.
In this problem, $a=0$, and $f(0)=0^2=0$.
$\lim_{x\to 0} f(x)$ requires looking at left/right limits, because of piecewise

Left limit: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^x = e^0 = 1$

Right limit: $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0^2 = 0$

Since $LL \neq RL$, the limit DNE, and f is <u>not</u> continuous at $x=0$.



Grade:
0.5 pt = attempt to apply correct definition of continuity at $x=0$
1 pt = find correct LL & RL and conclude limit DNE
0.5 pt = conclude f is discontinuous
1 pt = correct sketch of graph of $f(x)$

Exercise 35

$$f(x) = \begin{cases} cx^2+2x, & \text{if } x<2 \\ x^3-cx, & \text{if } x\geq 2 \end{cases}$$

Regardless of value of c, f is continuous when $x<2$, and when $x>2$, because it is a polynomial form. By continuity theorems, polynomials are continuous for all x.
Must check for continuity when $x=2$: $\lim_{x\to 2} f(x) = f(2)$

This requires $c(2)^2+2(2) = 2^3 - c(2)$

$$4c+4 = 8-2c \Rightarrow 6c=4 \Rightarrow c=\frac{4}{6}=\frac{2}{3}$$

When $c=\frac{2}{3}$, we get $\boxed{\lim_{x\to 2} f(x) = f(2)} = 8-\frac{4}{3} = \frac{20}{3}$

Thus f will be continuous on $(-\infty,\infty)$ if $\boxed{c=\frac{2}{3}}$.

Grade:
0.5 pt = state/argue that f is continuous on $(-\infty, 2)\cup(2,\infty)$ for any value of c
1 pt = know/show that at $x=2$, we require $cx^2+2x=x^3-cx$
1.5 pt = solve and state correct value of c.